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HIGH ENERGY PROPULSION SYSTEMS (HEPS) ANALYSIS

Robert T. Nachtrieb

OLAC-PL/RKFE
Edwards AFB, CA 93523-5000

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Final Report

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PHILLIPS LABORATORY
Propulsion Directorate
AIR FORCE MATERIALS COMMAND
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NOTICE

FOREWORD

This final report was submitted by Robert T. Nachtrieb. documenting work he performed while working at OLAC, Phillips Laboratory, (AFSC), Edwards AFB, CA, 93523-5000. OLAC PL Project Manager was Robert T. Nachtrieb.

This report has been reviewed and is approved for release and distribution in accordance with the distribution statement on the cover and on the SF Form 298.

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Deep space missions will require high specific impulse (I_{sp}) and large changes in velocity (Δv). The compact energy storage of nuclear fission systems and advanced fusion reactor concepts make them ideal power source candidates for future space applications. Fusion is particularly attractive because it produces more energy than fission, and does not produce long-lived radioactive waste. The purpose of this study was to produce a computer computer code to analyze the performance of fusion reactors as propulsion concepts. The code simplifies the task of comparing thruster performance while varying individual or multiple reactor parameters. Two potential candidate fusion reactor concepts were modeled and inserted into the reactor system model: (1) the Dense Plasma Focus (DPF); and (2) the Field-Reversed Configuration (FRC). For a fusion thermal rocket, the propulsion relations were established between fusion output and rocket performance, system mass, and mission capability. By varying fusion plasma temperature, fuel mixture ratio, and mission Δv, performance values for comparision are generated, including thrust, I_{sp}, jet power, jet specific power (α), thrust-to-weight ratio, and payload-to-system initial mass ratio.

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Nomenclature

a acceleration (m/s^2); local acoustic velocity (m/s)

A nuclear atomic mass (amu)

B magnetic flux density (T)

^{11}**B** fusionable isotope of boron

c speed of light in vacuum (m/s)

C_p specific heat (J/mole-K)

D deuterium, an isotope of hydrogen with one proton and one neutron

dc direct current (no oscillation)

e charge of an electron, (e= 1.602×10^{-19} C)

E energy release from fusion (MeV)

f ratio of electron to ion number densities ($f \equiv n_e/n_i$)

F applied force, thrust (N)

g gravitational constant (m/s^2); function of r_1, r_2, L, α', and β', used for coil thickness calculations; "Gaunt" factor for Bremsstrahlung production; Runge-Kutta intermediate derivatives

h Planck's constant, ($h = 6.626 \times 10^{-34}$ J-s)

h enthalpy of propellant (J/kg)

^{3}**He** helium isotope, with two protons and one neutron

^{4}**He** helium isotope, with two protons and two neutrons (alpha particle)

I current (A)

$\dot{I}$ time rate of change of current (A/s)

ICF Inertial Confinement Fusion

I_{sp} specific impulse (s)

k C_p/C_v; Boltzmann's constant ($k = 1.38066 \times 10^{-23}$ J/K); fractional density of fuel; DPF pinch compression ratio

K_e fraction of synchrotron radiation absorbed in chamber walls

kT energy, often measured in units of electron-volts (1 eV = 1.602×10^{-19} J)

l length of pinch

L axial length of the magnet (cm); inductance of plasma focus (H)

$\dot{L}$ time rate of change of plasma focus inductance (H/s)

^{6}**Li** fusionable isotope of lithium

$\dot{m}$ mass flow rate (kg/s)

$m(t)$ time varying mass (kg)

m mass (kg)

MCF Magnetic Confinement Fusion

n excitiation state of electron to be ionized; number density (cm^{-3})

N particle inventory

p pressure (Pa)

P power (W)

Q ratio out:in of power-or energy

r radius

R universal gas constant ($R = 8.314$ J/mole-K)

R_0 distance between centers of nuclei where the strong nuclear force becomes dominant

RR reaction rate between fuels (fusions/s-cm^3)

t time (e.g. DPF rise time)

T tritium, an isotope of hydrogen with one proton and two neutrons

T temperature (K)

U ionization potential (U = 13.6 eV for hydrogen); DPF capacitor bank voltage (V)

v velocity (m/s)

V volume (cm^3)

$\dot{W}$ work extracted from turbine ($\dot{W}_{turb} = P_{el,req}/\eta_{gen}$) (W)

x variable for Saha Equation

y fraction of charged particle energy deposited in plasma

Z atomic number of element

α jet specific power (kg/kW); helium-4 nucleus

α' ratio of outer to inner magnetic coil radius

β ratio of plasma kinetic pressure to magnetic pressure

β' quotient of coil length and twice the inner coil radius

γ power fraction

Δ difference, change

ϵ_0 permittivity of free space ($\epsilon_0 = 8.854 \times 10^{-12}$ F/m)

η efficiency

λ volume fraction of the magnet devoted to producing the field (around 90%); Bremsstrahlung wavelength ($\AA$)

μ_0 permeability of free space ($\mu_0 = 4\pi \times 10^{-7}$ H/m)

Φ neutron flux (neutrons /cm^2-s)

ρ density (g/cm^3); resistivity (Ω)

σ cross section of one particle to the other (cm^2); tensile strength (Pa)

$< \sigma v >$ average product of cross section and particle velocity (reaction rate parameter) (cm^3/s)

Σ effective cross section of LiH shielding (cm^2)

χ dimensionless ratio

Superscripts and Subscripts

0 initial condition

1,2 nozzle, radial locations

br bremsstrahlung radiation

c charged particle; chamber (FRC)

cone isotropic section shielded

cy cyclotron radiation

e electron

eff effective (atomic number)

el, req required electrical power

f final condition; fusion

fire firing of rocket motor

gen generator

hi high (e.g. high temperature - materials limited)

i ion

inj injected

j, k fuel elements

lo low (e.g. initial low temperature)

lp charged particles leaving the plasma

LiH lithium hydride

magnet from magnet

mat materials limited

m, opt maximum, optimum

mix mixed

n neutron; non-ionized

N particle (confinement in FRC)

ohmic Ohmic ($P_{ohmic} = I^2\rho$)

opt optimum

p power

payload payload demands

pin pinch (DPF)

plas plasma

prop propellant

r reactor; rise (e.g. rise time for DPF); radius (e.g. DPF pinch radius compression)

rd rundown (DPF)

s system; ship; separatrix (FRC)

shield shield

tank propellant, fuel tanks

ten tensile

therm thermal

turb turbine

vel velocity

z longitudinal

* "choked" condition at nozzle throat; dimensionless

α alpha particles (Helium-4)

θ azimuthal direction

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1 INTRODUCTION

The demands placed on space transportation systems for reduced mission time and increased payload capacity require the development of alternate and advanced space propulsion concepts. Deep space missions will require high specific impulse (I_{sp}) and large changes in velocity (ΔV). The compact energy storage of nuclear fission systems and advanced fusion reactor concepts make them ideal power source candidates for future space applications. Fusion is particularly attractive because it produces more energy than fission, and does not produce long-lived radioactive waste. Table 1 shows some possible energy sources for space propulsion. With the exception of antimatter annihiliation, fusion has the highest specific energy of any energy source.

Table 1: Comparison of Energy Sources

Energy Source	Specific Energy [J/kg]
Chemical (O_2/H_2)	1.5×10^{7}
Chemical (O_2/Be)	2.6×10^{7}
Atomic Hydrogen ($H+H=H_2$)	2.2×10^{8}
Metastable Helium (He^*)	4.6×10^{8}
Nuclear Fission (100%)	8.0×10^{13}
Nuclear Fusion (D-^{3}He)	3.5×10^{14}
Antimatter	9.2×10^{16}

The purpose of this study was to produce a FORTRAN computer code to analyze the performance of fusion reactors as propulsion concepts. The code simplifies the task of comparing thruster performance while varying individual or multiple reactor parameters. Shown in Fig. 1, a generalized fusion reactor system was modeled, establishing the power balance, fuel flow, power input, and reactor breakeven and ignition conditions.

Two potential candidate fusion reactor concepts were modeled and inserted into the reactor system model: (1) the Dense Plasma Focus (DPF); and (2) the Field-Reversed Configuration (FRC). The user selects one of the reactor types and the fusion fuel combination. The modular design of the program allows the user to add a subroutine with the model of a different reactor, if desired. For a fusion thermal rocket, the propulsion relations were established between fusion output and rocket performance, system mass, and mission capability. By varying fusion plasma temperature, fuel mixture ratio, and mission ΔV, performance values for comparision are generated, including thrust, I_{sp}, jet power, jet specific power (α), thrust-to-weight ratio, and payload-to-system initial mass ratio. The output file can be analyzed directly with text explanations or numeric columns.

1.1 Fusion Fundamentals

Fusion is the process that powers the stars. Fusion occurs when two light atomic nuclei join to form a heavier nucleus and a lighter nucleus with high kinetic energy. To achieve fusion, the distance between nuclei must be small enough for the short-range, strong nuclear attractive force to exceed the long-range Coulombic repulsive force. Fig.2 [1] shows the Coulomb potential barrier as a function of distance from the nuclear core, where Z_1 and Z_2 are the atomic numbers of fuel elements 1 and 2 respectively, e is the charge of an electron, and R_0 is the distance between the centers of the nuclei where the strong nuclear force becomes dominant. Fusion is easier with light elements because the nuclei have fewer protons, and therefore the Coulomb repulsion force is less. The trick then is to bring nuclei together with sufficient energy to overcome the initial repulsion.

Fusion reactions result in heavier nuclear products that are more stable than the light atomic nuclei reactants, because their binding energy is greater. Fig.3 [2] shows the change in binding energy per unit

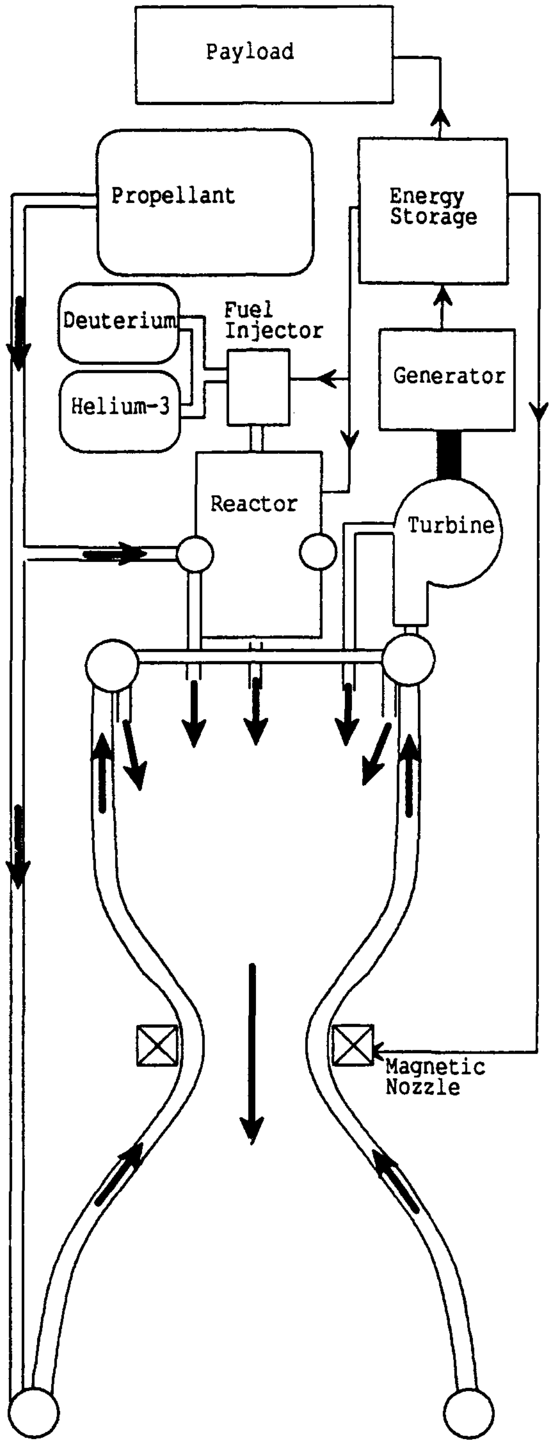

Figure 1: Fusion Rocket Schematic.

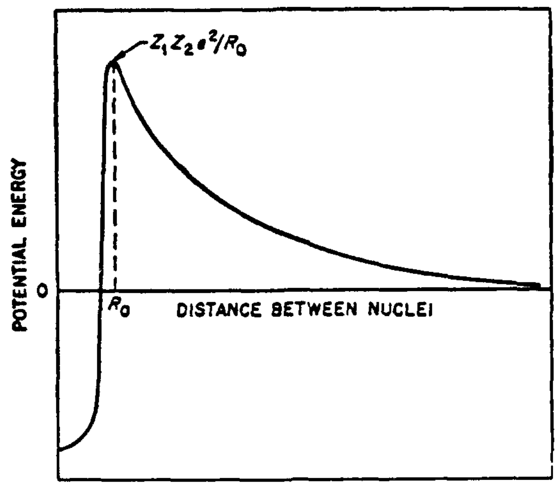

Figure 2: Coulomb Potential Barrier

atomic mass as a function of nuclear atomic mass. When two light elements fuse to form a heavier element, the the binding energy per unit atomic mass increases, leading to an increase in stability.

Fusion products have a combined rest-mass lower than the sum of their initial rest-masses. This mass difference is converted to kinetic energy of the products in accordance with the equation $E_f = \Delta m c^2$ [3], where $c = 3 \times 10^8$ m/s is the speed of light.

1.1.1 Fusion Fuels.

In a gas, fusion does not necessarily occur during each particle collision. The probability of a fusion reaction between two particles can be described in terms of a cross section, σ, of one fuel particle to the other. The cross section for particle systems in which one fuel element is stationary while the other impinges (target system) is different than the cross section for particle systems in which both fuel elements have the same average kinetic energy.[4] The cross section also depends on the mass of the particles, their atomic charge, the nuclear neutron-to-proton ratio, and the nuclear spin, but generally is highest for particles with low atomic charge and high energy. In choosing a fusion fuel, one must not only consider the engineering difficulty of fusing that fuel, but factors such as the fusion reaction products, materials limitations, and fuel availability.

The most commonly examined fusion reactions for light elements are shown in Table 2 [4,5,6,7]. The fusion fuel with the highest cross section at the lowest energy is an equal mix of deuterium and tritium. The temperatures required for significant rates of D-T fusion peak around 5 keV [6], or roughly fifty eight million degrees Kelvin. D-T fusion is the easiest to achieve, but the reaction gives 80% of its energy to neutrons, which carry no charge.[5] While adequate for ground-based power generation, the massive shielding necessary to protect the system components makes D-T a poor fuel choice for space applications.

Pure deuterium fusion (D-D) requires higher temperatures to achieve significant fusion rates and is therefore technologically more difficult, but as a benefit it releases only 34% of its energy to neutrons. In addition, a virtually inexhaustible supply of deuterium can be extracted from normal seawater, and the D-D reaction does not require the handling of radioactive tritium. Shown in Table 2 are the two branches of D-D fusion, which occur with almost equal probability. Although tritium is one of the products of D-D fusion, it can either be treated as waste or burned with deuterium ("catalyzed" D-D). More advanced reactions, such as p-^{6}Li and p-^{11}B produce no neutrons, but require temperatures as high as $kT = 100$ keV to produce significant fusion rates.[8]

D-^{3}He appears to be the most promising fusion fuel choice for propulsion. [9,10,11,12,13,14,15,16] The reaction products are protons and alphas, and at $kT \approx 30$ keV, the heating from fusion products equals the

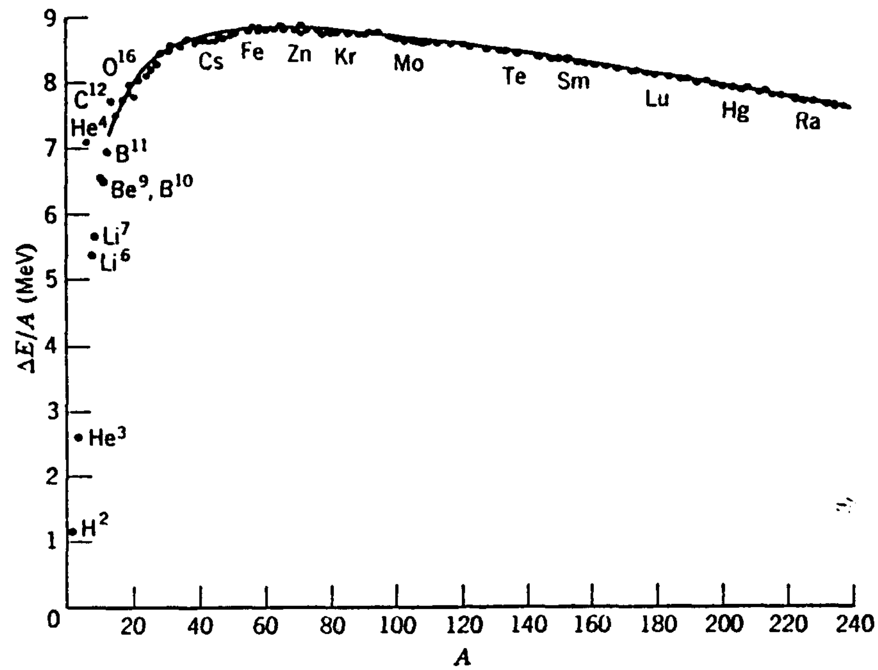

Figure 3: Rest-Mass Energy vs. Atomic Mass Number

Table 2: Standard (D-T) and Advanced Fusion Reactions

Name	Fusion Reaction (MeV); Abbreviated Form	Energy Release $E_{f,jk}$ (MeV;J)
D-T	D+T $\rightarrow \alpha(3.5)$ + n(14.1) T(d,n)^{4}He	$17.6; 2.82 \times 10^{-12}$
D-D(n)	D+D $\rightarrow {}^3$He(0.82) + n(2.45) D(d,n)^{3}He	$3.27; 5.24 \times 10^{-13}$
D-D(p)	D+D $\rightarrow$ T(1.01) + p(3.02) D(d,p)T	$4.03; 6.46 \times 10^{-13}$
D-^{3}He	D+^{3}He $\rightarrow \alpha(3.67)$ + p(14.68) ^{3}He(d,p)^{4}He	$18.35; 2.94 \times 10^{-12}$
p-^{6}Li	p+^{6}Li $\rightarrow {}^3$He(2.3) + $\alpha(1.7)$ ^{6}Li(p,α)^{3}He	$4.0; 6.41 \times 10^{-13}$
p-^{11}B	p+^{11}B $\rightarrow 3\ \alpha(2.88$ ea.) ^{11}B(p,3α)^{4}He	$8.64; 1.38 \times 10^{-12}$

radiative heat losses. Protons and alphas are charged particles, so their motion can be directed magnetically. With deuterium and helium-3 fusion, the only energy released to neutrons occurs from side, "parasitic" D-D reactions, and it may be possible to suppress the parasitic D-D and/or enhance the D-^{3}He fusion cross sections by aligning the spins of the nuclei[17,18,19,20,21]. Once D-^{3}He fusion has been achieved, the only remaining issue will be availability of ^{3}He. Currently this isotope of helium is rare on earth, and the only source occurs through the beta-decay of tritium. However, it appears that an economically feasible source of ^{3}He exists in the soil on the Moon's surface. [22,23,24,25] Mining of lunar ^{3}He might help spur along the development of space assets and controlled fusion power.

1.1.2 Reaction Rates and Power.

A plasma is a mixture of charged and neutral particles exhibiting collective behavior.[26] Plasmas are created by heating gasses to high temperatures such that gas molecules begin to dissociate and ionize. In a plasma with fuels j and k at ion densities n_j and n_k, and at energy kT, the "reaction rate" depends on the density of each element of the fuel and the cross section for the j,k combination. The average product of cross section and particle velocity, $<\sigma v>$, is specific for a given fuel at a given temperature. The fusion reaction rate is determined by [8]

$$RR_{j,k} = \frac{n_j n_k}{1+\delta_{j,k}} <\sigma v> \tag{1}$$

$$\delta_{j,k} = \begin{cases} 0 & j \neq k \\ 1 & j = k \end{cases} \tag{2}$$

Tables of $<\sigma v>$ values for particular reactions over a plasma temperature range are available.[4,5,6], and Fig. 4 shows $<\sigma v>$ for some of the reactions listed in Table 2. When the energy $E_{f,jk}$, released by a particular fusion fuel reaction, is multiplied by the reaction rate, one has the power density of the reaction which is a convenient figure for evaluating the usefulness of fusion fuels. When multiplied by the plasma volume, V_p, the power density gives total power produced by the fusion process. The fusion power is:

$$P_f = RR_{j,k} V_p E_{f,jk} \tag{3}$$

1.1.3 Radiation.

The two dominant types of radiation produced in a magnetically confined fusion plasma are bremsstrahlung and synchrotron.[3] Bremsstrahlung, which means "braking radiation" in German, occurs when charged particles collide with other charged particles and accelerate, emitting photons in the process. Synchrotron radiation is due to the gyration of electrons around magnetic field lines The total radiation power can be expressed [8] in terms of plasma temperature and magnetic pressure parameters as

$$P_{br} + P_{cy} = \left(c\sqrt{kT_e} + d\frac{1-\beta}{\beta}(kT_e)^2 K_e \right) n_i^2 V_p \ \text{[W]} \tag{4}$$

where

$$c = 5.35 \times 10^{-31} f_z \sum_j k_j Z_j^2 \tag{5}$$

$$d = 2.5 \times 10^{-32} \left(1 + \frac{T_i}{f_z T_e}\right)\left(1 + \frac{kT_e}{204}\right) \tag{6}$$

$$\begin{aligned} f_z &= \sum_j k_j Z_j \\ &= \frac{n_e}{n_i} \end{aligned} \tag{7}$$

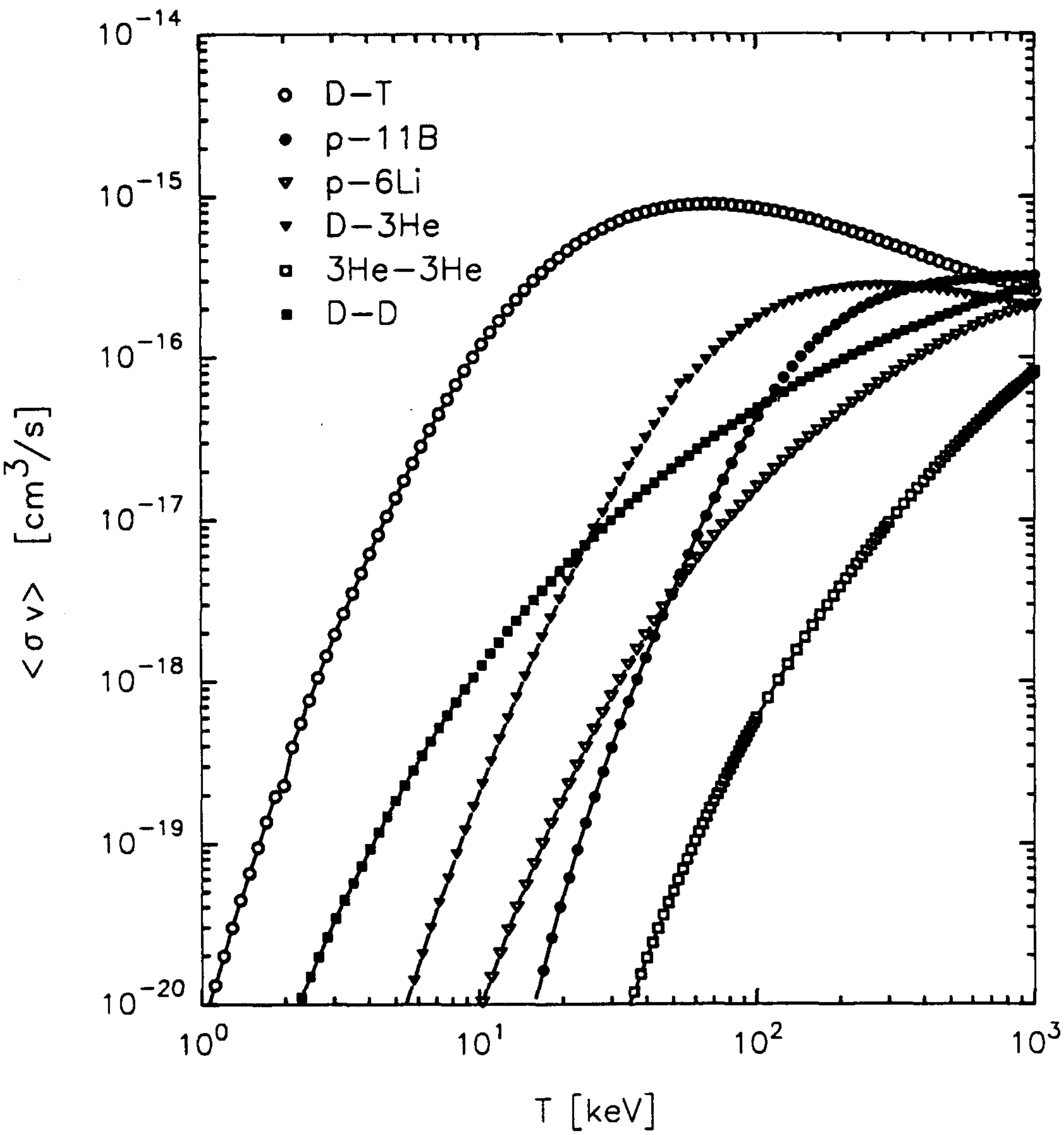

Figure 4: Reaction Rate Parameter for Common Fusion Reactions.

n_i [ions/cm^{-3}] is the ion density in the plasma; kT_e and kT_i [keV] are the respective electron and ion energies in the plasma; β is the ratio of plasma pressure to magnetic pressure; f_z represents the number of electrons present due to ionization of different elements, where k_j is the fractional density of fuel element j. K_c describes the fraction of synchrotron radiation absorbed in the chamber walls. (it depends on "plasma depth," involving system geometry, plasma electrical resistivity, reflectivity of the chamber walls, and plasma β.[8]) In high temperature fusion systems, synchrotron emmision is very large, and operation is not possible unless K_c is kept below about ten percent.

1.1.4 Magnetic Confinement

There are many different confinement schemes being developed to harness the energy of fusion. The three main types being studied are: magnetic (MCF); inertial (ICF); and electro-static. The Field Reversed Configuration and Dense Plasma Focus reactor designs considered as test cases are magnetic confinement schemes.

Controlled fusion is difficult to achieve because it requires maintaining very high temperatures and pressures. To achieve fusion by heating and containing the fuel, the plasma particles must have limited contact with the reactor's walls. Otherwise, the plasma is cooled when it transfers its kinetic energy to the walls.

The kinetic pressure of a plasma is the sum of the kinetic pressures of the charged particle components [7]:

$$p = \sum_j p_j = n_e kT_e + n_i kT_i + n_\alpha kT_\alpha \ldots \tag{8}$$

At equilibrium, the plasma kinetic pressure must equal the confining magnetic pressure, defined as $B^2/2\mu_0$.[7] For MCF, the magnetic field determines the plasma pressure, and thus the plasma density. The fusion reaction rate depends on the strength of the magnetic field indirectly, through the particle density and plasma temperature.

1.1.5 Ionization

All the particles leaving the fusion plasma have energies in the keV to MeV range, so it is very likely that they are completely ionized. The energies of ionization for the propellant can be calculated using the Bohr model for excitation [2],

$$\begin{aligned} U_i &= \sum_n \frac{\mu e^4 Z^2}{(4\pi\epsilon_0)^2 2h^2 n^2} \\ &= 13.6 \sum_n \frac{Z^2}{n^2} \text{ [eV]} \end{aligned} \tag{9}$$

where Z is the propellant atomic charge, and n is the state of each electron to be ionized. Monatomic hydrogen requires 13.6 eV to be ionized, so at keV and MeV temperatures hydrogen ionization will be virtually complete. However, if the hydrogen temperature drops, complete exhaust ionization becomes less certain.

Using a form of the Saha Equation for hydrogen [7], we can find the degree of ionization at lower temperatures. Setting the total heavy particle density equal to the ionized density plus the neutral particle density ($n_t = n_i + n_n$), we can find the degree of ionization (n_i/n_t) to be

$$\frac{n_i}{n_t} = \frac{(1+4x)^{1/2} - 1}{2x} \tag{10}$$

where x is defined as

$$x = \frac{n_t h^3 \exp(U_i/kT)}{(2\pi m_e kT)^{3/2}} \tag{11}$$

and h is Planck's constant, m_e is the electron mass, U_i is the ionization potential (13.6 eV for hydrogen), k is Boltzmann's constant, and T is the temperature. If kT is expressed in eV, then we can simplify x to

$$x = 3.313 \times 10^{-28} n_t (kT)^{-3/2} \exp(U_i/kT) \tag{12}$$

Fig. 5 shows the degree of ionization versus plasma temperature. For $n_t = 10^{15}$ cm^{-3}, ionization is essentially complete at a few eV.

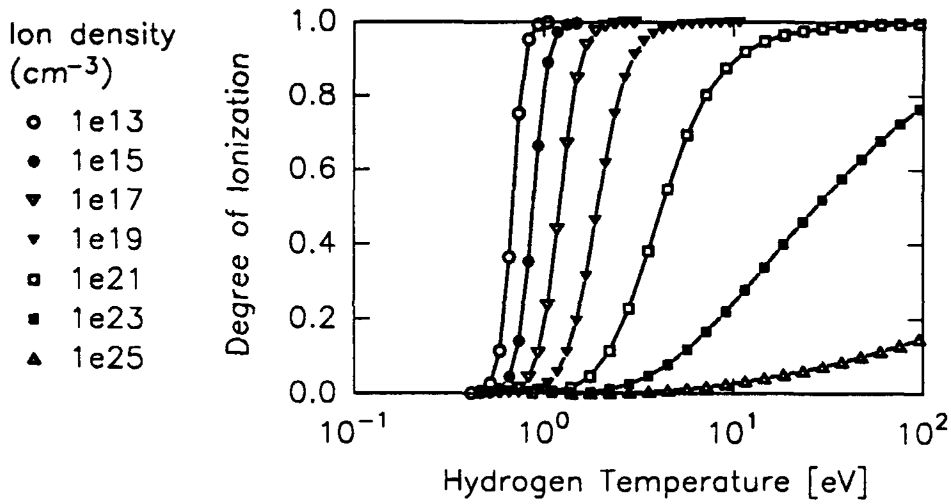

Figure 5: Ionized Fraction for Hydrogen, Based on Saha Equation.

1.2 Rocket Propulsion Fundamentals

A rocket provides thrust by Newton's Second and Third Laws and the principle of conservation of momentum. Newton's Second Law states that the acceleration, a, of a body with mass, m, is due to an applied force F ($F = ma$). Newton's Third Law states that every action must have an equal and opposite reaction. If propellant is accelerated in one direction by a propulsive force, the rocket feels the same force or thrust in the opposite direction. The rocket ejects propellant in the opposite direction of desired motion, and the momentum imparted to the propellant must be equal in magnitude but opposite in direction to the momentum imparted to the rocket, such that total momentum is conserved.

1.2.1 Rocket Equation

The change in velocity of a body is the integral of the body's acceleration over time. If a rocket of mass m exhausts propellant of mass dm at velocity v_e, the rocket velocity increases by dv. By conservation of momentum,

$$m dv = -v_e dm \tag{13}$$

Integrating Eq. (13) and raising both sides to powers of e gives the well known Rocket Equation [27]

$$\frac{m_0}{m_f} = \exp\left(\frac{\Delta v}{v_e}\right) \tag{14}$$

where the m_0 and m_f are the initial and final rocket masses, respectively.

The specific impulse ($I_{sp} = v_e/g$) of the rocket measures how efficiently the rocket uses its propellant to produce thrust. One wants the I_{sp} to be as high as possible since any extra mass carried aboard the rocket decreases the amount of payload allowed.

The Rocket Equation is used to determine how much of the rocket's total mass can be payload. It is most desirable to have the ratio m_0/m_f as close as possible to one, meaning the greatest possible fraction of the rocket is payload. Δv is essentially fixed for a given mission. Therefore, we need to make the denominator in the exponent of Eq. (14) to be large compared to the Δv, i.e. have a high I_{sp}. A propellant with low molecular weight is easier to accelerate to high exhaust velocities, thus improving the payload capabilities of the rocket.[28]

1.2.2 Nozzle Theory

The objective of a nozzle is to convert random thermal energy into directed kinetic energy. Knowing characteristics of the nozzle, such as exit-to-throat area ratio and length, allows us to describe thrust-generating ability.

There are several assumptions customarily made in deriving theoretical nozzle performance: (a) working fluid is homogeneous and invariant; (b) working fluid obeys ideal gas laws; (c) there is no friction; (d) flow is adiabatic; (e) propellant flow is steady and constant; (f) no reactions are occurring in propellant to change its composition or enthalpy; (g) all velocities exiting are axially directed, allowing one-dimensional analysis; (h) gas/plasma velocity, pressure, or density is uniform for any nozzle cross section.[29] These assumptions are idealizations of nozzle conditions, but are usually accurate to ten percent or less.

Formulas to be used include conservation of energy which, under the above assumptions, reduces to:

$$\left[h + \frac{v^2}{2}\right]_1 = \left[h + \frac{v^2}{2}\right]_2 \tag{15}$$

The ideal gas law: $p = nkT$; The isentropic flow process:

$$\frac{T_1}{T_2} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \tag{16}$$

where γ is the ratio of the specific heat for constant pressure C_p and the specific heat for constant volume C_V; and the acoustic velocity definition

$$a = \sqrt{g\gamma kT} \tag{17}$$

Using the assumptions and formulae listed above, the velocity at any point in the nozzle can be found in terms of the enthalpy difference of another point:

$$\begin{aligned} v_2 &= \sqrt{2(h_2 - h_1) + v_1^2} \\ &= \sqrt{\frac{2g\gamma}{\gamma-1} kT_0 \left[1 - \left(\frac{p_2}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} \end{aligned} \tag{18}$$

In this case, point number 1 has been chosen to be the inlet stagnation pressure and temperature, p_0 and T_0. As can be seen, the exhaust velocity v_2 reaches a maximum when the pressure ratio p_2/p_0 goes to zero, corresponding to an infinite expansion of the nozzle.

If a converging/diverging nozzle is employed, flow at the "throat" is "choked", indicated with a superscript *. The flow velocity at the throat, v^*, is equal to the local acoustic velocity a. the pressure and temperature at the throat correspond to the maximum isentropic mass flow. If the pressure ratio after the throat is decreased by expanding the nozzle, the propellant velocity exiting the nozzle, v_{ex} will exceed v^*, producing supersonic flow.

Thus, including each type of particle exiting the nozzle, the thrust of the nozzle is

$$F = \sum_i \dot{m}_i v_{ex,i} \tag{19}$$

1.2.3 Rocket Performance

The total system mass will consist of: the reactor; necessary shielding; propellant, fuel, structures and tanks; the radiator; the nozzle (magnetic or conventional); and the payload. Since everything but the payload is "overhead," it is desirable to reduce the system mass as much as practical to increase payload and/or decrease mission time.

Given the thrust, specific impulse, and initial mass, it is possible to determine the performance of the rocket. The thrust- to-weight ratio is $F/(m_0 g)$. The jet specific power, α, is defined as

$$\alpha = \frac{m_0}{P_{jet}} = \frac{m_0}{\frac{g}{2} F I_{sp}} \tag{20}$$

We define the starting mass, m_0, to be the sum of masses of the ship, m_s (which includes the payload, reactor, power system, etc.), the propellant, m_{prop}, and the propellant tank, m_{tank}.

$$\begin{aligned} m_0 &= m_s + m_{prop} + m_{tank} && (21) \\ m_f &= m_0 - m_{prop} && (22) \\ m_{prop} &= \dot{m}_{prop} t_{fire} && (23) \end{aligned}$$

For missions with long firing times and high propellant flow rates, the propellant tank mass can be large compared to the ship mass. If we define the tank fraction as $\chi_{tank} \equiv m_{tank}/m_{prop}$ and insert Eqs. (21), (22), and (24) into the Rocket Equation, Eq. (14), we can solve for the mission firing time as a function of Δv:

$$t_{fire} = \frac{m_s}{\dot{m}_{prop}} \frac{(e^{\Delta v/v_e} - 1)}{1 - \chi_{tank}(e^{\Delta v/v_e} - 1)} \tag{24}$$

Given the payload mass, some fraction of the ship mass, the payload mass fraction at launch is simply $m_{payload}/m_0$.

1.2.4 Magnetic Nozzle

Exhaust gases from a fusion rocket are likely to be partially or completely ionized, thus a magnetic nozzle[30,31,32] is an logical choice. Fig. 6 shows the magnetic field required for thrust conversion of different exhaust densities, as a function of temperature.

An azimuthal magnetic coil is one candidate for a nozzle substitute. However, there exist some engineering issues that must be resolved before operational used of magnetic nozzles is assured.[31]

1. At plasma temperatures below 1 eV, the plasma β must be below unity to prevent unacceptable amounts of resistive cross field mass transport (particles not following field lines); at 10 eV and β=1, this is not a problem.

2. In the temperature and particle density range we will most likely be operating (reactor-like conditions), nozzle generation of radiation can be severe [31] (on the order of a few MW/m^2 for a 1 m diameter nozzle throat and densities up to 10^{16} cm^{-3}) and have an prohibitive impact on wall-loading limitations and cooling requirements.

3. Due to the shape of the magnetic field lines, a "detachment" problem exists whereby particles follow the magnetic field lines around and deposit their energy back on the vehicle (negating any thrust benefit). One of the ways to counter the detachment problem is for the plasma entering the nozzle to have a field-free core, or similarly a region in which the core field lines doubles back on themselves.

Fig. 7 shows characteristics of a Meridional Magnetic Nozzle.[31]

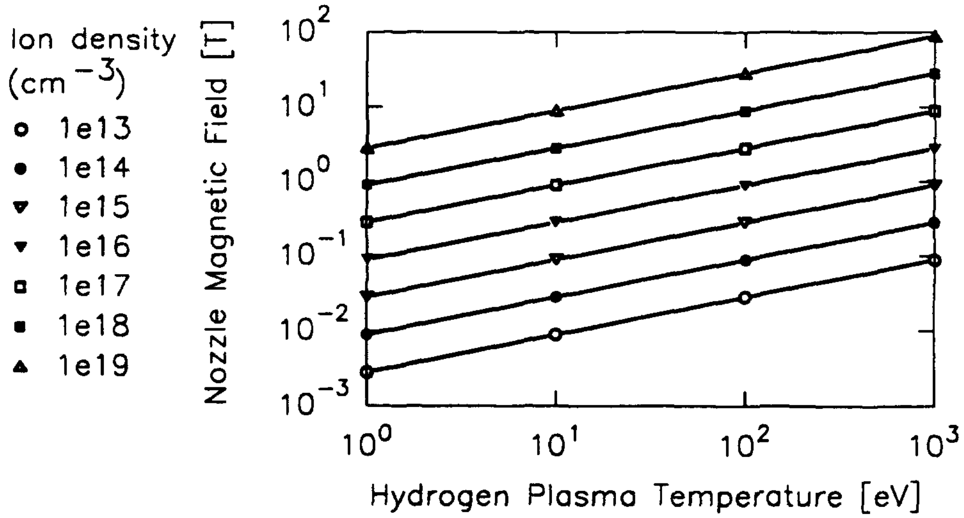

Figure 6: Magnetic Nozzle Field vs. Exhaust Temperature

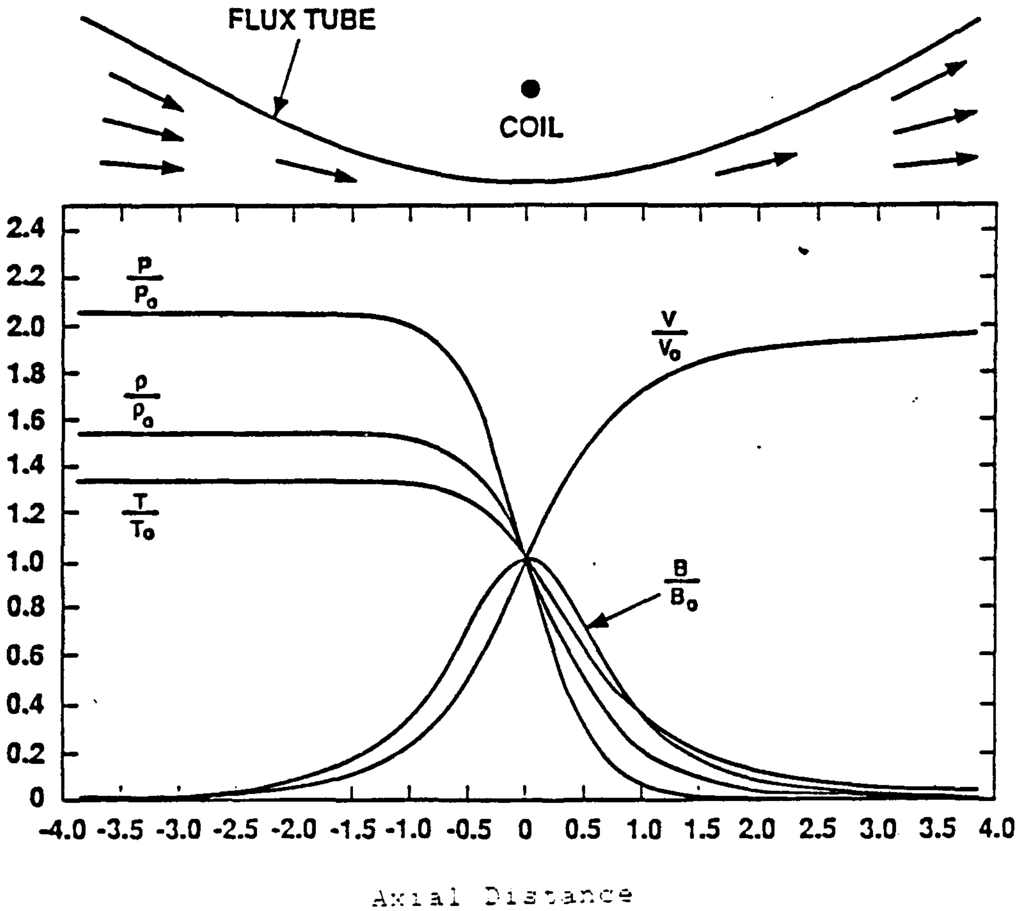

Figure 7: Meridional Magnetic Nozzle Characteristics

2 FUSION PROPULSION SYSTEM MODEL

2.1 Fusion Rocket

In the thruster design considered for this study, the propellant and the fuel are separate, so the lightest possible propellant, hydrogen, may be used. The high temperature plasma from the reactor is used to dissociate and ionize the diatomic hydrogen into protons and electrons. The degree of propellant ionization will depend on the final temperature and pressure of the propellant. If a large fraction of the propellant is ionized and the fusion fuel chosen such that the reaction products are charged particles, we can employ a magnetic nozzle to direct the charged particles and increase thrust.

Rocket systems employing separate propellant and fuels are usually limited by specific power (α) instead of specific impulse (I_{sp}).[33] Specific impulse-limited systems, such as conventional chemical rockets, are typically high-thrust vehicles, and since the fuel reaction products are the propellant, the amount of reaction energy imparted to each particle is fixed. With a specific power-limited system, more power may be added to the propellant particles until the power source becomes too heavy. Fusion promises to be an attractive propulsion power source because even small reactors should produce large powers.

A mathematical model of a fusion propulsion system is necessary to see the effects of changing one parameter versus another. The system model describes the use of the reactor's power, how the radiation and thermal heat is dissipated, method of electrical power generation, shielding requirements, thrust production, system mass, and rocket performance. It allows performance predictions and system performance optimizations.

2.2 System Description

Fig. 1 shows the component diagram of the fusion propulsion system considered in this study. The heart of the fusion propulsion system is the reactor, where the fusion fuel is burned and high temperature charged particles are produced. The reactor also produces electromagnetic radiation and neutrons that heat the walls and other components. Cold propellant cools these components by heat-exchange, then mixes with charged particles escaping from the fusion plasma. The heated propellant is expanded through a nozzle (conventional or magnetic, depending on the propellant temperature), providing thrust for the vehicle.

If neutronic reactions occur during the fusion process, shielding will be necessary to protect sensitive components and/or the payload. As the number of fusion neutrons produced is reduced, the mass of the shielding can be similarly reduced.

2.2.1 System Power Balance

The fusion propulsion system may be modeled in terms of its energy flow rates. A schematic of the system power balance is shown in Fig. 8.[8] Fusion power P_f is released by the fusion reaction within the plasma. The fusion power is distributed between charged particles and neutrons, P_c and P_n, respectively. A fraction, y, of charged particle power, P_c, is deposited in the plasma, heating the particles and electrons. If injected power P_{inj} is needed to sustain the reaction, the total output power is the sum of the fusion power, P_f, plus the injected power, P_{inj}.

The fusion plasma is cooled by energy transport outside the plasma. Energy losses include: (1) neutrons; (2) charged particles that do not deposit their energy in the plasma. Charged particles leaving the plasma can be heated fuel particles or energetic fusion products. These loss particles have power P_{lp}; and (3) unabsorbed radiation. Bremsstrahlung and cyclotron radiation powers, P_{br} and P_{cy}, respectively, are emitted by charged particles due to interaction with magnetic fields present. Thus the total radiation power comes at the expense of a reduction in kinetic energy of the particle flow. A fraction, γ_{cy}, of the cyclotron radiation is absorbed by the plasma, while bremsstrahlung radiation is assumed to be lost from the plasma. Although the injected power, P_{inj}, is meant to heat the fusion fuel or power the confinement system, it might also contribute to the radiation power loss as a side effect. For example, RF heating in the Field Reversed Configuration would increase particle collisions in the fusion plasma, increasing the bremsstrahlung radiation.

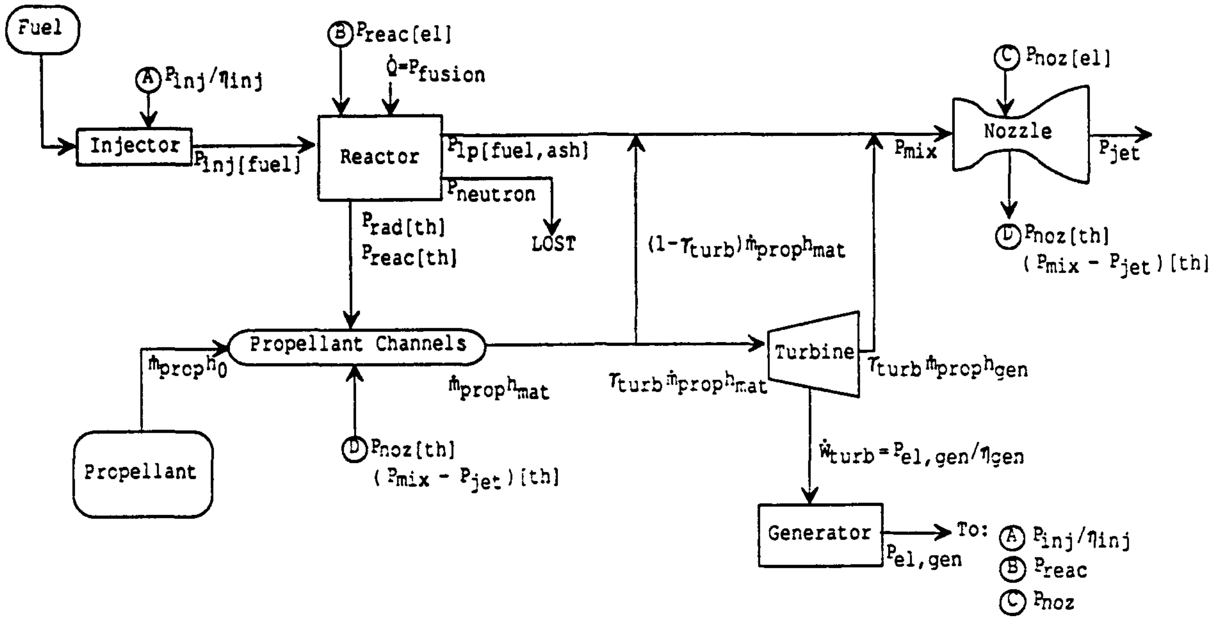

Figure 8: Rocket Power Balance

The required injected power, P_{inj}, is found by performing a power balance on the fusion plasma. This power can be imparted by charged or neutral beam injection, Ohmic heating, or radiowave heating.[7] In order to maintain a constant energy in the plasma, energy must be injected into it to make up for radiation and particle transport losses. Fortunately, some internal heating occurs in the plasma due to deposition of kinetic energy of charged particles, from the fusion reaction. If this internal heating is sufficient to make up for the losses, the plasma is said to be ignited; if not, external heating power is necessary. Thus, P_{inj} is sum all the power loss and heat addition mechanisms of the plasma. The heating of the plasma occurs with efficiency, η_{inj}, so the electrical power required to run the fusion reactor is P_{inj}/η_{inj}.

The power balance for the plasma is:

$$P_f + P_{inj} = P_{br} + (1 - \gamma_{cy})P_{cy} + P_{lp} + (1 - y)P_c \tag{25}$$

2.3 Propellant Heating

The propellant serves as a coolant that absorbs the thermal power from the rocket components. A minimum propellant flow is set that adequately cools the reactor, mixing chamber, and nozzle. The radiation and neutron power, along with Ohmic heating ($P_{ohmic} = I^2R$) generated by the magnetic nozzle (if present) and reactor, and the heat produced by the injector, constitute the thermal power, P_{therm}.

For cooling, the propellant must flow through passages near the hot component surfaces. The highest temperature the propellant can reach is limited by the material of the component being cooled. To maximize heat transfer and total energy absorption, the propellant should enter the passages at the lowest temperature possible. Assuming steady state, the thermal power, P_{therm}, absorbed by the propellant sets the necessary mass flow, according to:

$$\dot{m}_{prop} = \frac{P_{therm}}{h_{mat} - h_0} \tag{26}$$

where h_{mat} and h_0 are the final (materials limited) and initial enthalpies of the propellant.

2.4 Power Generation and Demands

A turbine extracts work adiabatically from the enthalpy of the heated working fluid. Conversion of this work to electrical energy occurs at an efficiency of η_{gen}. For a turbine-generator system, η_{gen} is a function of the temperature difference across the generator, while the power produced is a function of the enthalpy difference across the turbine. The electrical power produced must meet the demands of the payload, propulsion system components, and fusion reactor. The heated propellant, $\gamma_{turb}\dot{m}_{prop}$, sent to the turbines is sufficient to power the electrical generators ($0 \leq \gamma_{turb} \leq 1$). The remainder of the propellant, $(1-\gamma_{turb})\dot{m}_{prop}$, bypasses the turbines and is sent directly into the mixing chamber. The electrical power generated is

$$P_{el,gen} = \eta_{gen}\gamma_{turb}(h_{mat} - h_{turb})\dot{m}_{prop} \tag{27}$$

where h_{turb} is the enthalpy of the propellant exiting the turbines. After work has been extracted from the heated propellant in the turbines, the propellant is mixed with the escaping plasma particles (with power P_{lp}).

The power required for a magnetic coil, such as used in a magnetic nozzle or theta-pinch coil, depends on the required magnetic field, the resisitvity of the material, and the dimensions of the magnet. The power necessary to produce a magnetic field, B_z, in a single-turn solenoid is [7]

$$P_{magnet} = \left(\frac{B_z}{\lambda\mu_0 g(\alpha',\beta')}\right)^2 \rho_{res} r_1 \tag{28}$$

where ρ_{res} is the resistivity of the magnet, λ is the volume fraction of the magnet devoted to producing the field (around 90%), r_1 is the inner radius of the coil, and $g(\alpha',\beta')$ is a function of r_1, r_2, and L,

$$g(\alpha',\beta') = \left[\frac{\beta'}{2\pi(\alpha'^2-1)}\right]^{1/2} \ln\left[\frac{\alpha' + \sqrt{\alpha'^2+\beta'^2}}{1 + sqrt 1 + \beta'^2}\right] \tag{29}$$

where α' and β' are defined as $\alpha' = r_2/r_1$ and $\beta' = L/(2r_1)$, and L is the axial length of the magnet. The outer radius r_2 of the magnet depends on the radial thickness of the magnet, which in turn depends on the tensile strength, σ_{ten}, of the magnet material.

$$r_2 - r_1 = \frac{r_1}{\sigma_{ten}\left(\frac{2\mu_0}{B_z^2}\right) - \frac{1}{3}} \tag{30}$$

Ohmic heating of the magnets converts the electrical power, P_{magnet}, into thermal power of magnitude P_{magnet}. This contributes to the total thermal power, P_{therm}, and must be dissipated. The formula for P_{magnet} can be used to find the power demands of a magnetic nozzle or, in the case of a FRC, a theta pinch and end mirrors. If we employ superconducting materials for our magnets, we can drop the resistivity to zero, and thus the Ohmic cooling requirements will also drop to zero. Issues that remain to be examined include the superconductive temperature threshold, materials properties, and effects of neutron and cosmic radiation flux on superconductive properties. If we cannot use superconducting materials, another alternative is to reinforce the magnets with high-tensile strength steel bands, in effect increasing the tensile stength, σ_{ten}, of the magnet. However, the final model should also account for secondary gamma photons produced by neutron flux on high-Z materials, such as steel bands.

The payload power demands, P_{payld}, are dictated by the payload and mission; "housekeeping" power is around 40 KWe.[34] The total quantity of required electrical power is taken to be the sum of the magnet coils and/or capacitors, magnetic nozzle, pumps, power injectors, and payload.

$$P_{el,req} = \frac{P_{inj}}{\eta_{inj}} + P_{reac} + P_{payload} + P_{noz} \tag{31}$$

2.4.1 Reactor- and System-Ignition

Ideally, for the plasma to be self-sustaining, it must heat itself so that the injected power, P_{inj}, is minimized. To accomplish this, we must maximize the fractions of charged particle and cyclotron power, yP_c and $\gamma_{cy}P_{cy}$ respectively, that go to heating the plasma. Defining $Q_{p,r} \equiv P_f/P_{inj}$ as the ratio of fusion power over injected energy, we have reactor "breakeven" when $Q_{p,r} = 1$.[8]

If the generated electrical power, $P_{el,gen}$, is insufficient for the system needs, $P_{el,req}$, an external source $P_{ext} = P_{el,req} - P_{el,gen}$ is required. When the system requires no external power, we can establish that the system is "ignited": using $Q_{p,s} \equiv P_{el,gen}/P_{ext}$, ignition occurs when the system can supply its own electrical power needs. This is desirable because it excludes heavy battery or solar power supply systems from the rocket mass, improving performance. If ignition is achieved for the propulsion system, no external power is required ($P_{ext} = 0$), so $P_{el,gen} = P_{el,req}$ and $Q_{p,s} \to \infty$.

2.5 Plasma Mixing

The model assumes that, in the "mixing chamber", the fraction of heated propellant bypassing the turbine mixes with the fraction that passed through the turbine. The total propellant flow is then heated by the energetic charged fusion particles to a uniform temperature. This is one of the critical assumptions of this model. In other fusion propulsion studies, propellant-plasma mixing has been identified as a challenging area. In the McDonell Douglas-General Atomics study [10], the chamber in which plasmoids mixed with propellant was on the order of 50 m long to accommodate adequate thermal transfer.

The propellant mixes adiabatically and uniformly with the energetic plasma-loss particles escaping the fusion plasma, P_{lp}. Thus, the power of the mixed and heated propellant becomes

$$P_{mix} = P_{therm} + P_{lp} - \frac{P_{el,gen}}{\eta_{gen}} \tag{32}$$

Assuming complete ionization, with three degrees of freedom per particle, the heat capacity of the particles can be found by Fermi-Dirac statistics [35,36] to be $C_p = 5/2$ R J/mole-K, where $R = 8.3143$ J/mole-K is the universal gas constant. If we divide both sides by the average mass of a mole of particles (ions or electrons), we have C_p in units of [J/kg-K]. Assuming steady flow, we can find the stagnation temperature of the exaust by the identity

$$T_{mix} = \frac{P_{mix}}{\dot{m}_{out}C_p} \tag{33}$$

All the energy of the propellant is random, thermal energy instead of kinetic energy (stagnation). The exhaust is then directed through a nozzle, and the thermal power of the mixed propellant, P_{mix}, is converted to jet power.

2.6 Thrust Generation

The total mass flow rate exiting the rocket is the sum of the mass flow rates of the fuel and the propellant:

$$\dot{m}_{out} = \dot{m}_{prop} + \dot{m}_{fuel} \tag{34}$$

We can calculate the thrust generated by the propulsion system. Assuming that the exhaust temperature and density correspond to use of a magnetic nozzle, we can use Fig. 7 [31], which gives us state ratios for a meridional magnetic nozzle. We need the ratios of inlet temperature to throat temperature (T_{mix}/T^*), and exit velocity to throat velocity (v_{ex}/v^*). In Fig. 7 these ratios are listed as ($T_{z=-4.0}/T_{z=0}$) and ($v_{z=4.0}/v_{z=0}$).

By Eq. 33 we have the mixing temperature, T_{mix}, which is at the entrance to the nozzle. Dividing T_{mix} by (T_{mix}/T^*) gives the throat temperature of the exhaust. At the throat, the exhaust thermal energy is converted to kinetic energy:

$$\dot{m}_{out}C_pT^* = \frac{1}{2}\dot{m}_{out}v^{*2} \tag{35}$$

Expansion of the nozzle after the choke point accelerates the particles from v^* to v_{ex}. Solving for v^*, and multiplying by (v_{ex}/v^*) from Fig. 7, gives us the exit velocity of the exhaust. Thrust is then the product of the exhaust mass flow rate times the exit velocity.

2.7 Component Masses

2.7.1 Coils and Magnetic Nozzle

The reactor mass depends entirely on the type of reactor employed: if an FRC is used for fusion power, the theta coil and mirror magnets will be the dominant reactor mass; if a DPF is used, the reactor mass should be quite low. Similarly, the nozzle mass depends on whether magnetic coils are used or standard nozzle materials.

The main factor in determining mass of magnetic coils is the tensile strength of the material necessary to withstand the forces of magnetic pressure. With the thickness and the length of the cylindrical magnet, multiplying with the magnet material density gives the magnet mass. This technique will work for the theta pinch and mirrors of a FRC, and the magnetic nozzle of the FRC and DPF.

In Eqs. (28), (29), and (30), the radial thickness of a cylindrical magnet was found for a given inner radius, axial length, resistivity, tensile strength, and required field strength. Given the density of the magnetic material, ρ_{mag}, the mass of the magnet is the product of density and the cylinder volume:

$$m_{magnet} = \pi(2r_1\Delta r + (\Delta r)^2)L\rho_{mag} \tag{36}$$

where $\Delta r = r_2 - r_1$.

2.7.2 Shielding

The mass of the radiation shielding will depend on the duration of the mission, and the flux and type of radiation encountered. It is assumed that the reactor is the only source of man-made radiation, and that natural radiation includes trapped charged particles (Van Allen belts), Galactic Cosmic Radiation (GCR), and Solar Particle Events (SPE's or Solar Flares). While the positions of the Van Allen belts are known, and GCR maxima roughly follow an eleven year cycle, SPE's cannot currently be predicted. SPE doses can be severe, thus it is prudent to include on board an SPE "storm shelter". For a quick trip to Mars, a mass of five metric tons has been assigned to the storm shelter, based on estimated volume requirements and shield densities[37].

Since the reactor is running steady state, it is assumed the radiation dose rate is constant; it is further assumed that the total dose from the reactor to the occupants is the limiting factor in determining reactor shield thickness. Thus, for a given mission duration, the shield thickness must be varied to prevent the total dose from exceeding the maximum allowable dose. The dose depends on the energy of the attenuated neutrons as well as the flux.

Since we will most likely be employing the D(^{3}He,p)α reaction in our reactor [38], there will be an opportunity for parasitic D-D(n) reactions. There are two branches to D-D [3] with almost equal cross sections, as shown in Table 2. The products of the D-D reactions are all charged particles except for a 2.45 MeV neutron, coming from the D-D(n) branch; bremsstrahlung radiation produced is easily shielded against.[39] D-D fusion has a higher $<\sigma v>$ than D-^{3}He at almost all temperatures [8], so the D-D source of neutrons is not likely to be negligible. Thus, shielding against the effects of neutrons will be necessary.

A logical choice of shielding material for space-based applications is lithium-hydride (LiH) [40]; it is light-weight, has low-Z material to thermalize fast neutrons, and is a well- researched, established shielding material whose properties are known. Radiation shielding is a complex problem, so as a first approximation I have assumed a thick LiH shield ($\geq 40-50$ cm for $\approx 10^{13}$ neutrons/cm^{2}s) that attenuates collided neutrons completely based on the removal cross section, and those that pass through retain all their energy[1]. Neutrons

[1] Using this thick shield approximation, any secondary gammas that would be generated would exit the shield at the same rate as the uncollided neutrons, but at a lower energy rate; however, since we are using a low-Z shielding material, secondary

entering the shield collide with shield material, depositing some fraction of their 2.45 MeV, perhaps scattering, perhaps being absorbed. By assuming straight flux attenuation with a hydrogenous thick shield, a reasonable approximation [41,42,43] of actual shielding results is possible without having to perform numerical multi-group Monte Carlo calculations.

The source of the neutrons is the D-D(n) reaction, which proceeds at a reaction rate of $RR_{D-D(n)}$. Assuming isotropic distribution (the neutrons travel in all directions from the source uniformly), the original flux to the passengers is

$$\Phi_0 = \frac{RR_{D-D(n)} V_p}{4\pi r_{pass}^2} \tag{37}$$

We will use a spherical-shaped shadow shield (known as a "four-pi" shield) to block out a fraction, χ_{cone}, of the isotropic neutrons, taken here to be 1/8 of a sphere. The shield has inner and outer radii, r_1 and r_2, and to minimize shield mass for a given coverage, we wish to have r_1 as close to the source as possible. As a first approximation, the flux to the passengers is attenuated by the thick, hydrogenous shielding by the relation

$$\Phi = \Phi_0 \exp(-\Sigma \Delta r_{shield}) \tag{38}$$

where Σ is the effective cross section of the LiH shielding, given by

$$\begin{aligned} \Sigma &= n_{shield}(\sigma_{Li} + \sigma_H) \\ &= \frac{\rho_{shield} N_A v}{MW_{LiH}}(\sigma_{Li} + \sigma_H) \end{aligned} \tag{39}$$

Solving for the shield thickness, Δr_{shield}, we can calculate the shield volume by

$$V_{shield} = \chi_{cone} \frac{4}{3}\pi(r_2^3 - r_1^3) \tag{40}$$

which, when multiplied by the shield density, $\rho_{LiH} = 0.82$ g/cm^3, gives the shield mass.

We establish the final neutron flux, Φ, and therefore the shield thickness and mass, by the integrated dose allowable for the passengers/payload. From Chilton [39], ANSI and the NCRP listed 3.5×10^{-8} cSv-cm^2 for 2.5 MeV neutrons as the "prescribed neutron response functional datum for the phantom-related dose equivalent"; thus, given the total time of exposure to the flux, Φ, we can find the dose to the passenger. If we limit this dose to, say, 5 cSv annually[2], we can find the shield thickness, given the reactor temperature and thus the neutron source flux Φ_0. (See also 40CFR61 for exposure limits to radiation workers.)

gammas shouldn't be a problem. Design considerations for teh reactor first wall and surrounding components will have to include secondary gamma production

[2] Specified by US Nuclear Regulatory Commission (NRC), following recommendations from the National Council on Radiation Protection and Measurements (NCRP) [37]

3 REACTOR CONCEPTS

3.1 Dense Plasma Focus

The Dense Plasma Focus (DPF) [44], shown in Fig. 9 [45], is a coaxial electrode configuration, with the anode in the center and the cathode surrounding it. The simple geometry and compact size of the DPF make it attractive for fusion space propulsion applications.[46]

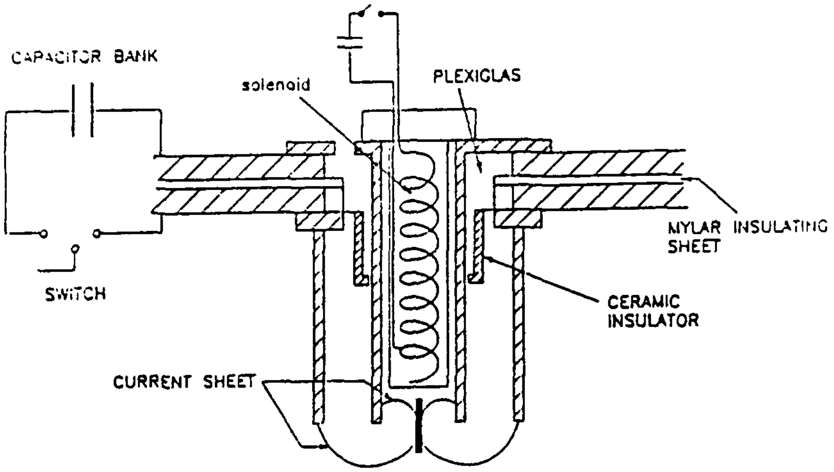

Figure 9: Dense Plama Focus Schematic, Mather Configuration.

The model used for the DPF is similar to that used by Leakeas.[47] A capacitor bank is discharged across the electrodes, ionizing the fusion fuel gas between them. Current flows radially between the electrodes, inducing an azimuthal magnetic field. The $J_r \times B_\theta$ force accelerates the ions and electrons axially. Motion of these particles in the "rundown" phase rapidly reaches steady state, using the "snowplow" model.[48]

The Dense Plamsa Focus (DPF) model consists of two parts: rundown and pinch. During the rundown phase, the gas between the coaxial Mather [44] geometry electrodes is ionized and a fraction swept forward (rundown mass) by the current sheath. The electrode lengths are set such that when the current sheath reaches the end, the current delivered by the capacitor banks is at a maximum.

In the pinch phase, the trapped plasma is constricted by the magnetic field field of the current at the end of the electrodes (maximum). The compressed plasma is heated like an ideal gas to fusion temperatures, with a Maxwellian energy distribution assumed. Fusion reactions occur at a rate corresponding to the density and temperature of the fuel plasma, with the accompanying energy release going to heat the rocket propellant.

Theory behind the DPF model is presented here. Appendix B gives some sample program input/output and the source code to the DPF circuit-equivalent program, `cir.c`.

3.1.1 Plasma Rundown

During the rundown phase, the DPF is approximated by an LC circuit, depicted in Fig. 10.

The time rate of change of the inductance, $\dot{L}$, is proportional to the plasma rundown velocity, experimentally observed to be roughly constant.[49] For devices with bank energy $W \approx 100$ kJ, the rate of inductance

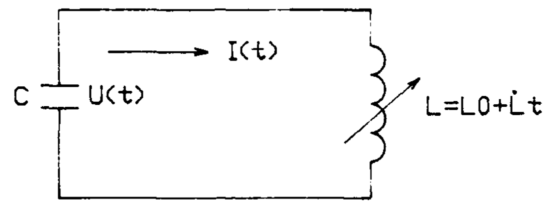

Figure 10: DPF Rundown Circuit

change $\dot{L} \approx 20$ mΩ. (H/s=Ω.) For a coaxial transmission line, the inductance change is

$$\dot{L} = v_{run}\frac{\mu_0}{2\pi}\ln\left(\frac{r_c}{r_a}\right) \tag{41}$$

where v_{run} is the rundown velocity, and r_c and r_a are the cathode and anode radii, respectively.

The circuit equation is

$$U - \dot{L}I - L\dot{I} = 0 \tag{42}$$

$$L = L_0 + \dot{L}t \tag{43}$$

where U is the capacitor bank voltage and I is the current delivered. Pinch and line resistance are neglected to simplfiy the circuit. If we hold the capacitor bank energy, W, the initial rate of current change, $\dot{I}_0$, and $\dot{L}$ constant, we can solve this circuit in dimensionless form:

$$U^* - I^* - (U_0^* + t^*)\frac{\dot{I}}{\dot{I}_0} = 0 \tag{44}$$

where

$$U^* = \frac{U}{(W\dot{I}_0\dot{L}^2)^{1/3}} \tag{45}$$

$$I^* = \frac{I}{(W\dot{I}_0/\dot{L})^{1/3}} \tag{46}$$

$$t^* = \frac{t}{(W/\dot{I}_0^2\dot{L})^{1/3}} \tag{47}$$

In terms of U^*, the circuit equation reduces to

$$U^* + \frac{2}{U_0^{*2}}\dot{U}^* + \frac{2}{U_0^{*2}}(U_0^* + t^*)\ddot{U}^* = 0 \tag{48}$$

which can be solved with a series solution of the form

$$U^*(t^*) = \sum_{n=0}^{\infty} a_n t^{*n} \tag{49}$$

$$a_0 = U_0^* \tag{50}$$

$$a_1 = 0 \tag{51}$$

$$a_{n+2} = -\frac{a_n U_0^*}{2(n+1)(n+2)} - \frac{a_{n+1}(n+1)}{U_0^*(n+2)} \tag{52}$$

This solution is a polynomial in t^*; the recursion relation for coefficients A_2 and above is also given.

The voltage on the capacitors is proportional to the charge, and the current delivered as a function of time is simply the negative time derivative of the capacitor charge.

$$I^* = -\frac{2}{U_0^{*2}}\dot{U}^* \tag{53}$$

Shown in Fig. 11 are the dimensionless voltage and current profiles as a function of time during the rundown, which follow expected trends. If we choose $U_0^* = 2.12$ [49], we get the maximum, optimum current of $I^* = 0.64$ with a rise time of $t^* = 1.45$.

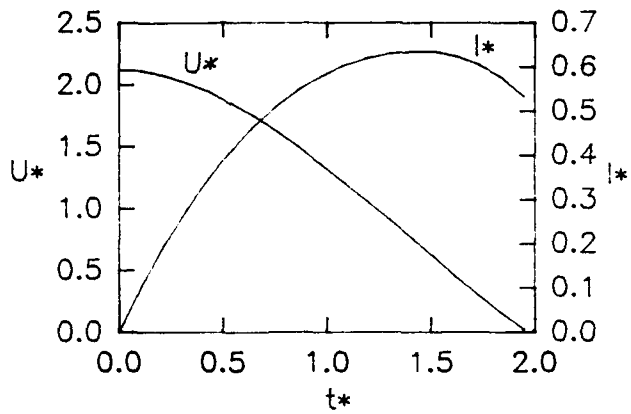

Figure 11: DPF Dimensionless Voltage and Current Profiles

3.1.2 Pinch

The pinch is approximated by a cylindrical plasma, whose dimensions remain constant during the stable pinch lifetime. When the pinch is in equilibrium, the magnetic pressure at the plasma edge must balance the kinetic pressure of the plasma (Bennett relation),

$$\frac{\mu_0 I^2}{8\pi^2 r_p^2} = nkT \tag{54}$$

where r_p is the pinch radius, $n = n_i + n_e$ is the particle number density in the pinch, and kT is the particle energy. The pinch becomes unstable when current is no longer delivered from the capacitors ($U = 0$), and thus the magnetic field drops to zero.

Pinch Dimensions. The pinch radius during the pinch, r_p, is taken to be [1]

$$r_p = r_0/k_r \tag{55}$$

where r_0 is the original pinch radius and k_r is the compression ratio. A compression ratio of $k_r = 10$ is conservative. We take the initial pinch radius to be the anode radius; it is actually less than the anode radius, but this is conservative for a given compression ratio.

$$r_0 \approx r_a \tag{56}$$

While compression ratios of $k_r = 10$ are common for Z-pinches, DPFs can have $k_r = 100$.[50]

From experimental scalings, we take the pinch length to be approximately the difference between the cathode and anode radii.

$$l_p \approx r_c - r_a \tag{57}$$

Pinch Lifetime. To simplify the DPF pinch physics, we assume that the pinch is stable for the time it takes the capacitors to discharge the rest of their energy at the maximum current and voltage at maximum current (dc). Theoretically, this estimate of pinch lifetime is a best-case scenario. The maximum current flows through the pinch, and the pinch is stable for as long as the capacitor bank has energy stored. Thus any subsequent calculations of energy yield from this pinch will represent an upper limit. However, compared to empircal scalings to high bank energies, the dc assumption gives conservative pinch lifetime predictions.

The capacitor bank energy devoted to rundown of the plasma is

$$W_{rd} = \int_0^{t_{max}} U(t)I(t)dt \tag{58}$$

where t_{max} is the time at which the current reaches a maximum. The remaining bank energy is devoted to the pinch:

$$W_{pin} = W - W_{rd} \tag{59}$$

The stable pinch lifetime is then taken to be

$$\tau_{pin} = \frac{W_{pin}}{U(t_{max})I(t_{max})} \tag{60}$$

Pinch Temperature. When we consider fusion heating and radiation cooling of the pinch plasma, we no longer have a pinch in equilibrium. For simplicity, it is assumed that the pinch dimensions remain constant over the stable pinch lifetime.[3] The initial plasma temperature is determined by the Bennett relation (Eq. 54), and the subsequent plasma temperature is determined by

$$\frac{d}{dt}NkT = U(t_{max})I(t_{max}) + P_{fusion} - P_{br} \tag{61}$$

where N is the particle inventory in the pinch, and P_{fusion} and P_{br} are the fusion and Bremsstrahlung powers that heat and cool the plasma.

3.1.3 Bremmstrahlung Production

The Bremsstrahlung emission spectrum is [1]

$$\frac{dP_{br}}{d\lambda} = 6.01 \times 10^{-36} \frac{g n_e^2 Z_{eff}}{\lambda^2 \sqrt{kT_e}} \exp\left(\frac{-12.40}{\lambda kT_e}\right) \tag{62}$$

where λ (Å) is the photon wavelength. The "Gaunt factor" g approaches $2\sqrt{3}/\pi$ at high plasma temperature. The peak Bremsstrahlung emission wavelength is

$$\lambda_{max} = \frac{6.20}{kT_e} \tag{63}$$

The total Bremsstrahlung power, P_{br} (W/m^3), is then

$$\begin{aligned} P_{br} &= \int_0^\infty \frac{dP_{br}}{d\lambda} d\lambda \\ &= 5.35 \times 10^{-37} Z_{eff} n_e^2 (kT)^2 \end{aligned} \tag{64}$$

[3] In fact, the pinch radius should not remain constant if there is net heating/cooling of the plasma, but the only alternative to this simplifying assumption is a 2-D time-dependent MHD code.

where kT (keV) is the plasma Maxwellian temperature, and n_e (m^{-3}) is the electron number density.The effective atomic number, Z_{eff}, is defined

$$\begin{aligned} Z_{eff} &= \frac{1}{n_e}\sum_i n_i Z_i^2 \\ &= \frac{n_D + n_{3He}2^2}{n_e} \\ &= \frac{5}{2} \end{aligned} \tag{65}$$

where the deuterium density, n_D, is equal to the helium-3 density, n_{3He}. Nitrogen plasma, which has a higher Z so gives a larger Bremsstrahlung contribution, but is not a fusion fuel, but has $Z_{eff} = 7$.

We approximate the Bremsstrahlung yield, E_br (J) by

$$E_{br} = V_p \int_{t_{max}}^{t_{max}+\tau_p} P_{br}\, dt \tag{66}$$

where V_p is plasma volume (m^{-3}), τ_{pin} (s) is stable pinch time, l_{pin} (m) is pinch length.

3.1.4 Issues

There are issues to be addressed and assumptions to be validated before the DPF can be employed in a propulsion concept.

- Particles in the pinch are thermalized, their energy distribution is Maxwellian, and that Maxwellian reaction rate parameters ($<\sigma v>$) may be used. In fact, the cross sections in the pinch may not be entirely (or at all) Maxwellian, but some combination of beam-target and Maxwellian. Until the true temperature-cross section relationship in the pinch is known, there will be a degree of uncertainty in specifying the fusion output power.

- The $kT \propto I^2$ scaling relation is valid up to fusion temperatures. More accurate models of the Dense Plasma Focus exist [51,52,53], including time-dependent, three-fluid MHD computer simulations. This model of the DPF used is rudimentary, but scaling of the plasma focus device has not yet been experimentally verified for currents greater than $I \approx 1$ MA or energies greater than $kT \approx 1$ keV.[54]

- Pinch lifetimes, which are on the order of 150 ns [55], can be made orders of magnitude longer, through the use of embedded magnetic fields that prevent instability modes. Alternately, capacitors or other power supplies can be made to fire at very high rates of fire ($\nu \approx 10^4\ s^{-1}$) to compensate.

- The pinch is a cylindrical plasma with no internal magnetic field, whose initial dimensions are measured; the nature of the time rates of change of the pinch energy and dimensions is calculable. The short time scales and complicated plasma behavior of the pinch make accurate theoretical or experimental models difficult to realize.

- Fusion fuel and charged fusion products are confined during pinch lifetime; it follows that all fusion energy to charged particles is confined (absorbed) in the pinch plasma during pinch lifetime. Experimental observations indicate that large particle emissions occur during plasma focus pulses [56,57,58], but this model assumes that particle transport has been minimized for each pulse.

- Electrode materials can be made to withstand the pulsed, multi-mega ampere currents delivered.

- Power supply specific masses (energy per unit mass) can be enhanced so supply masses are not prohibitively large for space applications.

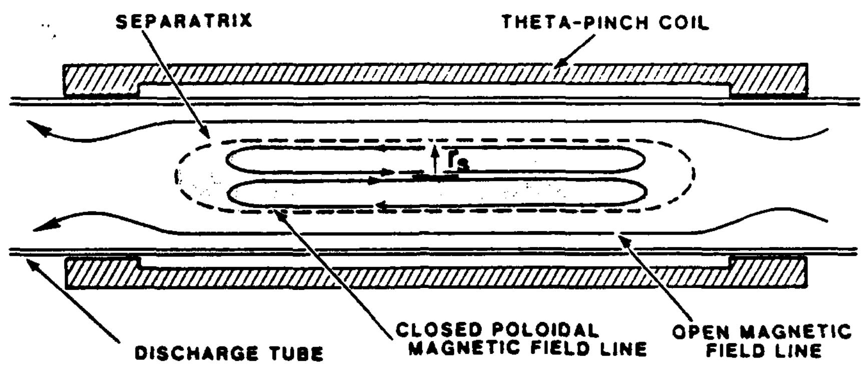

Figure 12: Field-Reversed Configuration

3.2 Field-Reversed Configuration

Fig. 12 shows a cross section of a Field Reversed Configuration.[59] It is a member of the Compact Toroid (CT) family, characterized by torus-shaped plasmas without enclosing magnetic coils or core. The FRC is stretched along the center axis (prolate), has little or no toroidal field, and has high beta, β. The FRC has many appealing features for space propulsion:

- The design of the FRC burn reactor is a simple cylindrical tube, simplifying construction and maintenance, enhancing reliability, encouraging modular design, and reducing cost.
- The FRC reactor does not have a solid core, as does the tokamak, and translation of the FRCs has been demonstrated.[60] Thus it is possible to separate the formation and burn chambers to optimize the characteristics of both.
- The FRC's open field lines act as a natural diverter to exhaust burnt fuel (ash) and heat the propellant. The open field lines that surround the FRC would permit, in steady state operation, the flow of propellant to mix with the hot plasma boundary and heat the propellant.
- Since and the open field lines travel out the end of the theta pinch coil, we can take advantage of the simple geometry of the FRC by having several FRC burn chambers in line, with the open field lines flowing through each of them.
- FRC's seem ideally suited for rocket use in conjunction with a magnetic nozzle. The field free core and sharp field reversal more easily permit charged particles to detach from the magnetic field lines and escape from the nozzle.[7,59]

Although FRCs demonstrate macrostability, they are subject to microinstabilities and have life-times on the order of 300 μs.[61] The $n = 2$ rotational instability has recently been eliminated by using weak multipole fields after FRC formation.[59] Ultimately, it is hoped that the FRC will burn in steady-state; the FORTRAN code assumes steady state FRC operation and refueling, but a more conservative case would be to find the pulse-averaged fusion power, as is done for the DPF.

FRC Formation. FRCs are commonly generated by field-reversal of a single-turn, elongated solenoid (theta pinch). FRC generation can be described in four stages [61]: pre-ionization, field-line connection, heating, and containment.

Methods for preionization include Z-pinch injection and conical theta-pinch discharge at the ends, but the most common method is discharging a theta pinch coil. The fill gas in the formation chamber is turned into a cold plasma during the preionization stage, freezing the bias magnetic field direction. Once the bias

is established in the plasma, the current in the theta coil is reversed, reversing the direction of the magnetic field near the coil surface. The field direction in the center remains the same as the original bias. The magnetic field lines are connected at the ends of the theta pinch coil, containing the fusion plasma. Field line reconnection is traditionally induced by passive or independent fast-trigger end mirrors. Finally, the closed field lines contract axially and come to equilibrium with the fields from the end mirrors; adiabatic contraction heats the plasma to the burn temperature.

Reactor Scaling. Studies concerning FRC lifetime scaling [62]-[80], reflect the desire for steady state operation. Particle transport out of the closed-field line region is a prominent factor in limited configuration lifetimes. A typical particle is contained within the separatrix for an average time, τ_N, which is defined as the particle inventory divided by the loss rate. It may not be possible to contain particles absolutely, but it is desirable to make τ_N as large as possible. As we increase reactor size, temperature, and particle density to reactor conditions, an important issue is to see how the confinement time scales with these parameters.

A critical issue to the validity of the FRC thruster model is scaling to higher temperatures. Empirical scaling laws have been fitted to experimental data [65,66,71,72,76,78,79,80], but there is indication[59] that the results of particle and flux confinement and configuration lifetime are very dependant on seemingly unrelated factors as fill gas pressure, pre-ionization technique, and reactor dimensions.

At this stage of FRC research, effort is directed towards demonstration of the concept through prolonging confinement and lifetime.[59] Ultimately the FRC reactor would operate at steady state conditions, with fuel injection to compensate for particle loss and fuel burn-up.[38,81] There are currently no experimental data of FRC operation at reactor-like conditions, and so the model relies upon theoretical[67,68,69,70,73] and experimental[77] predictions.

Steinhauer[77] suggests that at reactor conditions particle confinement would scale classically with temperature and linearly with size (effective diffusivity inversely proportional to size). Krall's[69] proposed scaling law is based on low-frequency drift wave transport, agreeing with other theoretical and experimental results in that as the collision frequency decreases, energy and particle loss also decreases.

Miley's Velocity Space Loss Sphere (VSLS) model[67,68] represents the upper bound of particle confinement in the FRC. It assumes that the dominant loss mechanism is due to scattering into a region of (H, P_θ) space, or velocity space, where particles are not contained; other mechanisms, such as instability or cross-field diffusion are negligible, i.e. have been solved. (H and P_θ are the total energy and canonical angular momentum of the particle considered, respectively.) VSLS confinement represents the best confinement possible because it implicitly assumes having overcome current engineering challenges. A fraction, between one and ten percent, of the VSLS scaling law is used to represent particle confinement. This allows the model to provide optimistic forecasts for the FRC's potential, but prevents the model from overestimating the the FRC's ability.

Table 3 summarizes features of these four models.

Table 3: FRC Particle Confinement Time Scaling: $\tau_N \sim l_s^a x_s^b r_c^c n^d T^e$

Scaling	a	b	c	d	e
TRX-2 Experiment[59,77]	0	1.0	1.0	0.9	1.5
VSLS Theory[73]	0	2.7	1.8	-1.0	0.2
Krall Theory[69,70]	0	2.0	2.0	0.5	-0.5
Lower Hybrid Drift Theory[59]	0.6	3.3	2	0.5	-0.7

3.2.1 Power Balance

The FRC is assumed to run in steady state, so the temperature in the plasma remains constant. The power balance for the plasma may be written

$$\frac{\partial}{\partial t}E_{plas} = P_{inj} + P_f - P_n - P_{lp} - (1-\gamma_{cy})P_{cy} - P_{br} = 0 \tag{67}$$

where γ_{cy} is the fraction of synchrotron radiation power absorbed in the plasma. Assuming Maxwellian cross sections apply, the fusion power, P_f, is defined as

$$P_f = \sum_j RR_j E_{f,j} V_{plas} \tag{68}$$

where the fusion power is divided up between neutrons, P_n, and charged particles, P_c. For D-^{3}He, $P_c/P_f \approx 0.97$.[8] The the plasma volume, V_{plas}, is assumed to be defined by the separatrix boundries, $V_{plas} = \pi r_s^2 l_s$, where r_s and l_s are the separatrix radius and length, respectively.

For a FRC, the plasma beta is[59]

$$<\beta> = 1 - \frac{x_s^2}{2} \tag{69}$$

$$x_s = r_s/r_c \tag{70}$$

This can be used to calculate the bremsstrahlung and synchrotron radiation, $P_{br} + P_{cy}$, described in Eq. (4).

The particle transport power out of the FRC plasma, P_{lp}, can be expressed

$$P_{lp} = \frac{N_i k T_i (1 + f_z r_t}{\tau_N} + (1-y)P_c \tag{71}$$

where y is the fraction of the fusion charged particle power, P_c, that is absorbed in the plasma, and τ_N is the particle confinement time, scaling for which is found in Table 3.

3.2.2 Issues

Restart Capability. For a deep space mission, it may be necessary to coast for long periods of time or power down the reactors. Given the current difficulty in achieving ignition in D-T fusion reactors, as well as the higher plasma temperatures necessary for D-^{3}He fusion, the issue of remote restart capability deserves consideration. Several options exist:

1. If employing multiple fusion reactors to produce thrust, one reactor could be kept "lighted" at all times to act as a "pilot"[82] for the other reactors. Fusion fuel consumption might prohibit keeping even one ignited fusion reactor burning at all times.

2. Cold restarts could be made at all times with use a conventional space-based nuclear fission reactor, such as a SP-100[83], as a starter motor. If this approach is taken, additional shielding requirements would have to be taken into account.

3. Missions within the solar system might permit use of solar panels to trickle charge high specific-energy capacitor banks; these would be discharged to ignite a chemical[84] or D-T fusion "ramp"[85], which would in turn ignite the D-^{3}He reactor(s). Mass of necessary solar panels and capacitor banks, if necessary, might be prohibitive.

Quenching. If the open field lines are to be used to carry the propellant around the FRC, where mixing with the plasma edge heats the propellant, too much propellant might overcool the FRC, initiating instabilities or taking the FRC out of the ignited regime. Thus, there might exist a maximum amount of propellant allowable to be heated directly by the FRC, depending on the FRC dimensions and stabilization methods.

Unburnt Fuel Recovery. Due to the scarcity of ^{3}He sources, and the high particle-loss rate of an FRC, unburnt ^{3}He should be recovered before it is exhausted from the nozzle; recovery apparatus will add to the system mass, but it might be less expensive to the mission than exhausting costly fuel.

One possible method to retain unburnt fuel would be to convert the entire system to nuclear-electric propulsion instead of nuclear-thermal: using the venetian blind type direct conversion (dc) method[8], fusion power could be converted to electricity, which would in turn power a magneto-plasma-dynamic (MPD) or arcjet propulsion device. Recovery of the ^{3}He from the dc device would then be simplified. However, the multiple conversions required for nuclear-electric propulsion would likely be less efficient than direct nuclear-thermal.

The objective of recovering the unburnt fuel is to reinject it into the plasma for reburning, to increase the efficiency with which the ^{3}He is used. If the reactor type is an FRC, multiple steady-state CTs could be maintained, separated by mirror coils, analogous to the series tandem-mirrors in the SAFFIRE concept[86]; the open field lines would flow through each chamber in series, carrying with them unburnt fuel and ash (burnt fuel); these open field lines act as natural diverters[59], and in successive chambers the particle flow could be skimmed off, the ^{3}He separated and reinjected. Using this method, propellant would only flow around the last FRC, as (1) no unburnt fuel diverting would be possible, and (2) propellant injected earlier in the series would pollute the separation process. Since translation of FRCs has already been demonstrated[60,79], one formation chamber could be used for all FRCs, which would then be sent to their respective burn chambers.

4 Example Problem

An example problem illustrates the main features of this model, using the DPF as the fusion reactor.

We start with a capacitor bank energy $W = 10$ MJ, an initial rate of current change $\dot{I}_0 = 1$ MA/μs, and a rate of inductance change $\dot{L} = 20$ mΩ. (1 H/s = 1 Ω) This gives the characteristic voltage, A_U, current, A_I, and time, A_t:

$$A_U = (W\dot{I}_0\dot{L}^2)^{1/3} = 159 \text{ kV} \tag{72}$$

$$A_I = (W\dot{I}_0/\dot{L})^{1/3} = 7.94 \text{ MA} \tag{73}$$

$$A_t = (W/\dot{I}_0^2\dot{L})^{1/3} = 7.94\ \mu\text{s} \tag{74}$$

Using $I^* = U^* = 0.64$ and $t^* = 1.43$, the maximum current, voltage at maximum current, and the rise time are

$$I_m = 5.08 \text{ MA} \tag{75}$$

$$U_m = 102 \text{ kV} \tag{76}$$

$$t_r = 11.4\ \mu\text{s} \tag{77}$$

The energy of the capacitor banks is divided between the rundown and the pinch: $W = W_{rd} + W_{pin}$. The fraction of bank energy devoted to rundown is

$$\frac{W_{rd}}{W} = \int_0^{1.43} U^* I^* dt^* = 0.89 \tag{78}$$

so the rest goes to the pinch:

$$W_{pin}/W = 0.11 \tag{79}$$

This compares favorably with experimental results, e.g. Gates[87] gets $W_{pin}/W \approx 0.12 - 0.24$.

We assume the pinch is stable for the time it takes the capacitor banks to deliver the remainder of energy at voltage $U = U_m$ and current $I = I_m$. Although this might not be accurate (e.g. pinch might be stable only as long as current increases), the assumption conserves capacitor bank energy, and gives a conservative pinch lifetime. Noting that

$$A_U A_I A_t = W \tag{80}$$

the pinch lifetime is

$$\tau_{pin} \approx \frac{W_{pin}}{U_m I_m} \tag{81}$$

$$= \frac{0.11W}{(0.64A_U)(0.64A_I)} \tag{82}$$

$$= \frac{0.11}{(0.64)^2} A_t \tag{83}$$

$$= 2.13\ \mu\text{s} \tag{84}$$

This order of magnitude compares with Herold[88], who gets $\tau_{pin} = 11\ \mu$s at $I = 20$ MA.

The initial plasma temperature is set by

$$nkT = \frac{\mu_0 I^2}{8\pi^2 r_{pin}^2} \tag{85}$$

where

$$n = N/\pi r_{pin}^2 l_{pin} \tag{86}$$

$$r_{pin} = r_0/k_r \tag{87}$$

$$l_{pin} \approx r_c - r_a \tag{88}$$

Taking

$$
\begin{aligned}
r_c &= 8 \text{ cm} && (89)\\
r_a &= 3 \text{ cm} && (90)\\
N &= 6 \times 10^{19} && (91)\\
I &= I_m && (92)
\end{aligned}
$$

the plasma radius, volume, density, and initial temperature are

$$
\begin{aligned}
r_{pin} &= 0.428 \text{ mm} && (93)\\
V_{pin} &= 2.88 \times 10^{-2} \text{ cm}^3 && (94)\\
n &= 2.08 \times 10^{21} \text{ cm}^{-3} && (95)\\
kT_0 &= 6.71 \text{ keV} && (96)
\end{aligned}
$$

For now we assume that the fraction of fusion fuel burnup during the pinch lifetime is small compared to the fuel inventory, so that we can treat the fuel density as constant. (We check this after calculating the fusion yield.) Rather than doing several iterations by computer, we get a rough idea of the fusion and radiation yield by doing one Runge Kutta (RK) iteration by hand.

Whereas the Euler method integrates the temperature by

$$kT(t_0 + h) \approx kT(t_0) + gh \tag{97}$$

where g=dkT/dt is the local "slope", RK integrates using the weighted average of four Euler slopes:

$$
\begin{aligned}
g_1 &= \left.\frac{d}{dt}kT\right|_{kT_0} && (98)\\
g_2 &= \left.\frac{d}{dt}kT\right|_{kT_0+g_1h/2} && (99)\\
g_3 &= \left.\frac{d}{dt}kT\right|_{kT_0+g_2h/2} && (100)\\
g_4 &= \left.\frac{d}{dt}kT\right|_{kT_0+g_3h} && (101)
\end{aligned}
$$

The RK estimate is then

$$kT(t_0 + h) \approx kT(t_0) + \left(\frac{g_1}{6} + \frac{g_2}{3} + \frac{g_3}{3} + \frac{g_4}{6}\right) h \tag{102}$$

We start with the the time rate of change of the plasma energy:

$$\frac{d}{dt}kT = \frac{1}{N}\left(U_m I_m + P_f - P_{br}\right) \tag{103}$$

The power delivered from the capcitor banks is

$$
\begin{aligned}
U_m I_m &= (102 \text{ kV})(5.08 \text{ MA}) && (104)\\
&= 0.516 \text{ MJ}/\mu\text{s} && (105)
\end{aligned}
$$

The fusion heating power is

$$
\begin{aligned}
P_f &= W_f V_{pin} \frac{n^2}{4} < \sigma v > && (106)\\
&= (18.3 \text{ MeV})(1.602 \times 10^{-13} \text{ J/MeV})(2.88 \times 10^{-2} \text{ cm}^3) && (107)\\
&\quad \frac{(2.08 \times 10^{21} \text{ cm}^{-3})^2}{4}(< \sigma v > \text{ cm}^3/\text{s}) &&\\
&= 9.14 \times 10^{16} < \sigma v > \text{ MJ}/\mu\text{s} && (108)
\end{aligned}
$$

Similarly, the radiation cooling is

$$\begin{aligned}
P_{br} &= AZ_{eff}V_{pin}n^2\sqrt{kT} && (109)\\
&= (5\times10^{-31})(2.5)(2.88\times10^{-2}\ \mathrm{cm}^3)(2.08\times10^{21}\ \mathrm{cm}^{-3})^2(kT\ (\mathrm{keV}))^{1/2} && (110)\\
&= 0.156kT^{1/2}\ \mathrm{MJ}/\mu s && (111)
\end{aligned}$$

Our function reduces to

$$\begin{aligned}
\frac{d}{dt}kT &= \frac{1}{6\times10^{19}}(0.516+9.14\times10^{16} <\sigma v> -0.156kT^{1/2}) && (112)\\
&\quad \frac{\mathrm{keV}}{1.602\times10^{-16}\ \mathrm{J}}\frac{10^6\ \mathrm{J}}{\mathrm{MJ}} \\
&= (64.4+9.51\times10^{18} <\sigma v> -16.2kT^{1/2})\ \mathrm{keV}/\mu s && (113)
\end{aligned}$$

Table 4 lists some D-^{3}He $<\sigma v>$ values for convenient plasma temperatures.

Table 4: Useful $<\sigma v>$ values.

kT (keV)	$<\sigma v>$ (cm^3/s)
1	2.1×10^{-26}
10	3.0×10^{-19}
100	1.3×10^{-16}
1000	2.5×10^{-16}
5000	2.0×10^{-16}

Using $h=\tau_{pin}=2.13\ \mu s$, we approximate the temperature change as shown in Table 5

Table 5: RK Temperature Change Approximation.

t	kT (keV)	$g\equiv dkT/dt$ (keV/μs)	
t_0	6.71	12.0	$=g_1$
$t_0+h/2$	19.5	21.1	$=g_2$
$t_0+h/2$	29.1	96.0	$=g_3$
t_0+h	211	2040	$=g_4$

At the end of the pinch lifetime ($t=0+\tau_{pin}$), the plasma temperature is approximately

$$\begin{aligned}
kT(\tau_{pin}) &\approx 6.71(\mathrm{keV})+\frac{1}{6}(12.0+2(21.1)+2(96.0)+2040)(\mathrm{keV}/\mu s)(2.13\ \mu s) && (114)\\
&= 818\ \mathrm{keV} && (115)
\end{aligned}$$

We take the average temperature to be the temperature at $t=\tau_{pin}/2$, which is $kT=412$ keV. We then approximate the fusion and bremsstrahlung energy yields by multiplying the pinch lifetime by the powers at the average temperature:

$$\begin{aligned}
E_f &\approx \tau_{pin}P_f(kT_{av}) && (116)\\
E_{br} &\approx \tau_{pin}P_{br}(kT_{av}) && (117)
\end{aligned}$$

This gives $E_f\approx 53.1$ MJ, and $E_{br}\approx 6.74$ MJ.

We now can check our assumption that the fusion fuel burned is much less than the original inventory. The number of fuel ions burned, N_f, is twice the pulse fusion energy divided by the energy per fusion

reaction:

$$
\begin{aligned}
N_f/N &= \frac{2E_f/W_f}{N} && (118)\\
&= \frac{2(53.1\text{ MJ})}{(18.3\text{ MeV})(1.602\times10^{-13}\text{ J/MeV})(6\times10^{19})} && (119)\\
&= 0.603 && (120)
\end{aligned}
$$

So we have $N_f < N$, as we should, but we don't have $N_f << N$, which we need for our assumption that $n \approx$ constant during the pinch lifetime. However, once the fuel density drops enough to affect the reaction rate, most of the fuel has burned up already. Thus for this example problem, we are relatively safe as long as $N_f < N$.

If we fire at a repetition rate of $\nu = 1\text{ s}^{-1}$, our time averaged fusion and radiations powers are

$$
\begin{aligned}
\bar{P_f} &= E_f\nu && (121)\\
\bar{P_{br}} &= E_{br}\nu && (122)
\end{aligned}
$$

We set the propellant flow rate to absorb thermal power produced in the rocket. The thermal power is the sum of the input (capacitor) and output (fusion, radiation, nozzle ohmic heating) powers. Here we take this to be

$$P_{th} = (W + E_f + E_{br})\nu + P_{noz} \qquad (123)$$

P_{noz} depends on the nozzle cooling requirements, i.e. the current flowing through the magnetic nozzle, which depends on the magnetic field required for the degree of ionization of the propellant. For now we assume that P_{noz} is negligible compared to the radiation and injected powers, and will check this later.

$$
\begin{aligned}
P_{th} &\approx (W + E_f + E_{br})\nu && (124)\\
&= (10 + 53.1 + 6.74)\text{ MW} && (125)\\
&= 69.8\text{ MW} && (126)
\end{aligned}
$$

The propellant flow required to absorb this thermal power depends on the temperature increase we can give the propellant, which depends on the temperature limitations of the materials conducting the propellant. We assume the propellant starts at $T_{lo} = 50$ K, and the walls, turbine blades, etc., limit the propellant high temperature to $T_{hi} = 3000$ K. This corresponds to an enthalpy difference of $(h_{hi} - h_{lo}) = 105$ MJ/kg. Thus we find the mass flow of propellant by

$$
\begin{aligned}
\dot{m}_{prop} &= \frac{P_{th}}{h_{hi} - h_{lo}} && (127)\\
&= \frac{69.8\text{ MJ/s}}{105\text{ MJ/kg}} && (128)\\
&= 0.665\text{ kg/s} && (129)
\end{aligned}
$$

We divert the heated propellant to a turbine-generator system, which charges the capacitor banks. If we assume a combined efficiency of turbines and generators of $\eta_{gen}\eta_{turb} = 0.5$, the power of the propellant after the turbines is

$$
\begin{aligned}
P_{prop} &= P_{th} - \frac{P_{el}}{\eta_{gen}\eta_{turb}} && (130)\\
&= 69.8\text{ MW} - \frac{10\text{ MW}}{0.5} && (131)\\
&= 49.8\text{ MW} && (132)
\end{aligned}
$$

We know the propellant mass flow rate and the energy flow rate. We can do a quick check to see if the propellant is ionized. The energy per atom is roughly the propellant power divided by the mass flow rate:

$$kT_{prop} \approx \frac{P_{prop}}{\dot{m}_{prop}} \tag{133}$$

$$= \frac{(49.8\ \text{MW})}{(0.665\ \text{kg/s})} \frac{(1.67 \times 10^{-27}\ \text{kg/atom})}{(1.602 \times 10^{-19}\ \text{J/eV})} \tag{134}$$

$$= 0.781\ \text{eV} \tag{135}$$

Since hydrogen has almost no ionization at $n \geq 10^{19}\ \text{m}^{-3}$ and $kT \approx 1$ eV, we can use a conventional nozzle instead of a magnetic nozzle. Thus we do not need to cool the ohmic heating of a magnetic nozzle's coils, and thus we can neglect P_{noz}.

Using nozzle characteristics that compare propellant properties at the turbine exit, nozzle throat, and nozzle exit,

$$\chi_{therm} \equiv \frac{kT_{prop}}{kT_{thr}} \tag{136}$$

$$\chi_{vel} \equiv \frac{v_{ex}}{v_{thr}} \tag{137}$$

we can find the thrust and specific impulse of the rocket. Taking

$$\chi_{therm} = 1.35 \tag{138}$$

$$\chi_{vel} = 2 \tag{139}$$

we get $kT_{thr} = 0.578$ eV. The throat converts the energy increase to kinetic energy:

$$kT_{thr} - kT_{prop} = \frac{m}{2} v_{thr}^2 \tag{140}$$

$$v_{thr}^2 = \frac{2(0.781 - 0.578)\ \text{eV}(1.602 \times 10^{-19}\ \text{J/eV})}{1.67 \times 10^{-27}\ \text{kg/atom}} \tag{141}$$

$$v_{thr} = 6.12\ \text{km/s} \tag{142}$$

The exit velocity is

$$v_{ex} = (2)(7.65\ \text{km/s}) \tag{143}$$

$$= 12.2\ \text{km/s} \tag{144}$$

The thrust and specific impulse are then

$$F = \dot{m}_{prop} v_{ex} \tag{145}$$

$$= (0.665\ \text{kg/s})(12.2\ \text{km/s}) \tag{146}$$

$$= 8.14\ \text{kN} \tag{147}$$

$$I_{sp} = \frac{v_{ex}}{g} \tag{148}$$

$$= \frac{12.2\ \text{km/s}}{9.8\text{m/s}^2} \tag{149}$$

$$= 1250\ \text{s} \tag{150}$$

Note that while I_{sp} is independent of ν, F is directly proportional to ν.

We estimate the mass of the ship to be the sum of the payload and the capacitor banks:

$$m_s = m_{payload} + m_{cap} \tag{151}$$

$$m_{cap} = \frac{W}{\rho_{cap}} \tag{152}$$

where we take $\rho_{cap} = 2$ kJ/kg to be the specific mass of the capacitor banks. Taking $m_{payload} = 10^5$kg, we get

$$m_s = 10^5 \text{ kg} + \frac{10 \text{ MJ}}{2 \text{ kJ/kg}} \tag{153}$$

$$= 1.05 \times 10^5 \text{ kg} \tag{154}$$

If we assume a tank fraction of $\chi_{tank} = 0.1$, the maximum possible velocity change is $\Delta v_{max} = 29.4$ km/s. If we choose a velocity change, we get the firing time from Eq. (24):

$$\Delta v = 25 \text{ km/s} \tag{155}$$

$$t_{fire} = 37.3 \text{ days} \tag{156}$$

From Eqs. (21) and (20), we can get the initial rocket mass, m_0, and jet specific power, α_j. We can also calculate the payload mass fraction at launch:

$$m_0 = 2.46 \times 10^6 \text{ kg} \tag{157}$$

$$\frac{m_{payload}}{m_0} = 4.06\% \tag{158}$$

$$\alpha = 49.4 \text{ kg/kW} \tag{159}$$

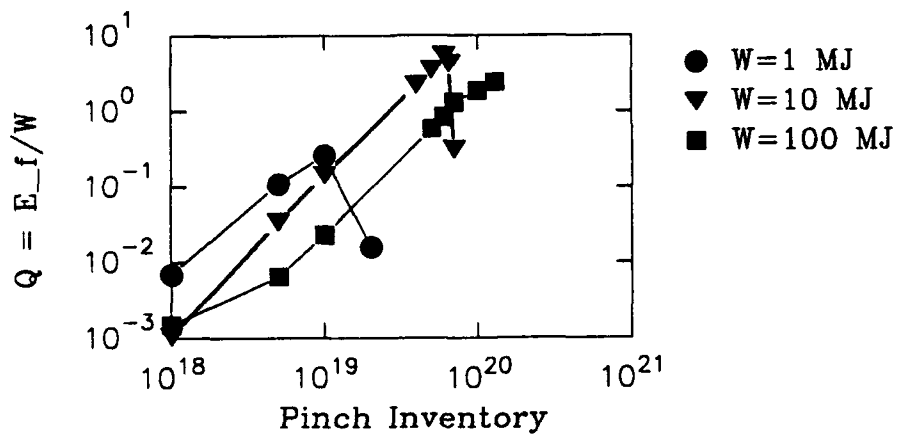

Figure 13: DPF Q vs. Pinch Inventory, N.

Figure 13 compares DPF $Q \equiv E_f/W$ values for different bank energies, W, as a function of the number of fusion fuel atoms in the pinch, N. For $W = 1$ MJ, the the DPF fusion relative output peaks around $N = 10^{19}$, but below breakeven ($Q < 1$). At high pinch inventories, the current compression gives low initial pinch plasma temperatures, kT_0, so the fusion reaction rate parameter, $< \sigma v >$, is low. At low pinch inventories, the current compression gives kT_0 beyond the fusion $< \sigma v >$ peak, so the fusion rate is again lower. At the inventory corresponding to the Q peak, the average plasma temperature during the pinch lifetime is at the $< \sigma v >$ peak.

The peak Q increases as the inventory ***and*** the bank energy increase, but for two reasons:

1. The increased bank energy gives a higher maximum current, I_m, which overheats the initial plasma temperature, kT_0. If the inventory is increased, the average plasma temperature can be brought down to the $< \sigma v >$ peak.

2. Equations (74) and (84) show that the pinch lifetime scales as $W^{1/3}$. If the pinch plasma burns longer at the peak fusion rate, the Q increases.

For $W = 10MJ$,the DPF achieves $Q = 5.30$ at $N = 6 \times 10^{19}$. At higher bank energies, the peak Q levels off: the pinch lifetime is long enough, and the fusion reaction rate high enough, that all the fusion fuel burns.
Using the following input criteria,

$$W = 10 \text{ MJ} \quad (160)$$
$$\dot{I}_0 = 1 \text{ MA}/\mu\text{s} \quad (161)$$
$$\dot{L} = 20 \text{ m}\Omega \quad (162)$$
$$N = 6 \times 10^{19} \quad (163)$$
$$r_c = 8 \text{ cm} \quad (164)$$
$$r_a = 3 \text{ cm} \quad (165)$$
$$\chi_{tank} = 0.1 \quad (166)$$

Figures 14, 15, and 16 compare rocket performance as a function of the firing frequency, ν, of the DPF.[4] The hollow symbols indicate the lighter payload mass, $m_{payload} = 5 \times 10^3$ kg (roughly the mass of a satellite), and the filled symbols indicate the heavier payload, $m_{payload} = 10^5$ kg (heavy payload). The circles indicate a small firing velocity change, $\Delta v = 10$ km/s (roughly the Δv to get to the moon), and the triangles indicate a high velocity change, $\Delta v = 25$ km/s (roughly the Δv to get to Mars).

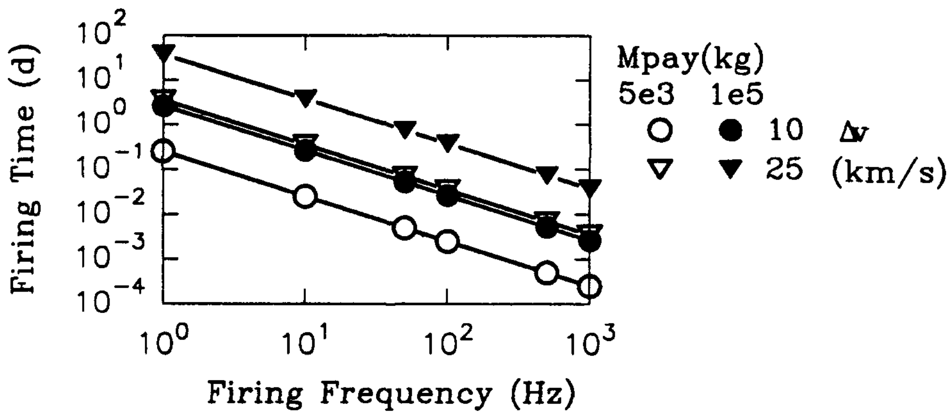

Figure 14: Total Firing time vs. Firing Frequency, ν

As one would expect, the low mass, low velocity change missions have the shortest firing times, lowest jet specific powers, and the highest thrust-to-weight ratios.

[4] For systems where more than one DPF fires at the same time, the firing frequency would be the product of the number of DPFs and the firing frequency of one DPF.

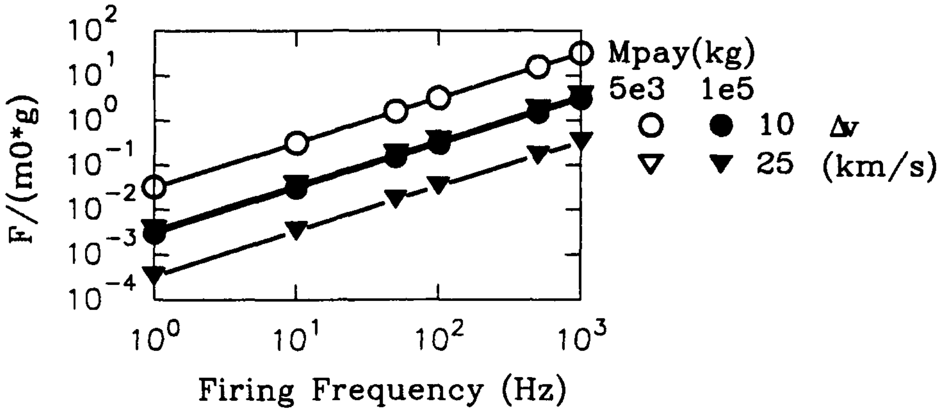

Figure 15: Initial Thrust to Weight Ratio vs. Firing Frequency, ν

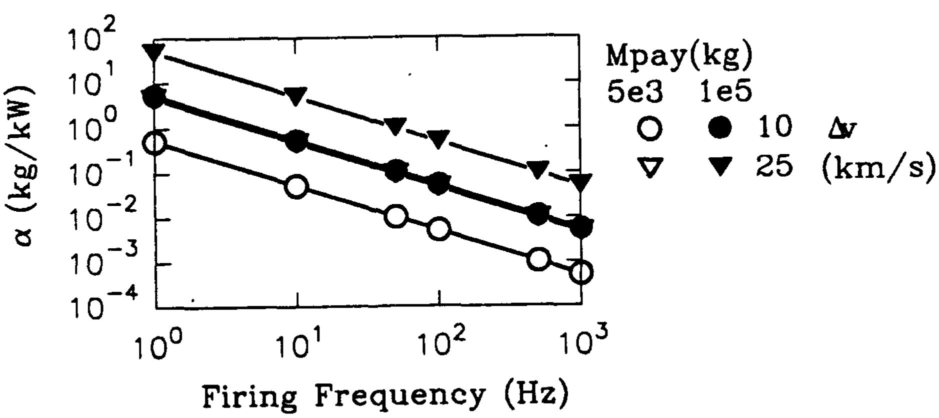

Figure 16: Jet Specific Power, α, vs. Firing Frequency, ν

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A FORTRAN source code

A.1 heps.for

```
C23456789                                                                   0
      PROGRAM heps

      INCLUDE comblk.for

*****************************************************************
C   begin main program
      WRITE(*,*)'High Energy Propulsion System (HEPS) 0.99a 06/92'
      WRITE(*,*)'R.Nachtrieb: OL-AC PL/RKFE;UIUC Fusion Studies Lab.'
      WRITE(*,*)'Reactor models: DPF,FRC'
      WRITE(*,*)

C     read in parameters from .hep data files
      CALL getvar
      WRITE(*,*)'Incrementing variable: ',IncVar
      WRITE(*,193)' lower,upper,howmany ',lower,',',upper,',',howmany

*****     BEGIN CALCULATIONS     *****
C   preloop calculations conserve computing time by reducing work
C   to do in main loop.
      CALL preloop

      IF(loglin.EQ.1)THEN
C       increment IncVar LOG10
        IF(howmany.EQ.1) THEN
          inc=LOG10(upper)-LOG10(lower)
        ELSE
          inc=(LOG10(upper)-LOG10(lower))/(howmany-1)
        ENDIF
        IF(IncVar.EQ.'T')operate=LOG10(lower)
        IF(IncVar.EQ.'deltav')operate=LOG10(lower)
      ELSE      !otherwise increment IncVar linearly
        IF (howmany.EQ.1) THEN
          inc=upper-lower
        ELSE
          inc=(upper-lower)/(howmany-1)
        ENDIF
        IF(IncVar.EQ.'T')operate=lower
        IF(IncVar.EQ.'deltav')operate=lower
      ENDIF

**********     BEGIN LOOP     **********
      DO 40 count=1,howmany,1
        bomb=0

      IF(loglin.EQ.1)THEN        !increment IncVar LOG10
        IF(IncVar.EQ.'T')T=10**operate
        IF(IncVar.EQ.'deltav')deltav=10**operate
      ELSE       !otherwise increment IncVar linearly
```

```
        IF(IncVar.EQ.'T')T=operate
        IF(IncVar.EQ.'deltav')deltav=operate
      ENDIF

          Ti=T/(1+rt) ![keV]
          Te=rt*Ti

C       plasma power balance, thermal powers for reactors
        IF(reactype.EQ.2)THEN
          CALL pow_d
          CALL therm_d
c        ELSEIF(reactype.EQ.3)THEN
c          CALL pow_f
c          CALL therm_f
        ENDIF

C       system power recirculation, cooling,
        CALL cooling

C       rocket performance
        CALL rocket
        IF(bomb.NE.0)GOTO 39

C       reactor, system masses
        CALL mass
*****     END CALCULATIONS     *****

C       display output
        CALL output

C       goto here if have to skip iteration (robust)
39      CONTINUE

        operate=operate+inc
40    CONTINUE
**********      END LOOP      **********

      END
C     main program roc
****************************


      BLOCK DATA
c   define constants in accordance with FORTRAN77 syntax
C   constants
      REAL mu0,k,g,pi,J_MeV,J_keV,u,elec,Uion,MWelec,remnflux,
     &rhoLiH,MWLiH,Wddn,Wddp,Wd3He,Wp11B,Wp6Li,Wdt,W23He,Wtt

C   constants (from DATA blocks)
      COMMON/CONST/mu0,k,g,pi,J_MeV,J_keV,u,elec,Uion,MWelec,remnflux,
     &rhoLiH,MWLiH,Wddn,Wddp,Wd3He,Wp11B,Wp6Li,Wdt,W23He,Wtt
```

```
C   constants
      DATA mu0,k,g,pi/1.2566E-6,1.38066E-23,9.8062,3.141592654/
      DATA J_MeV,J_keV,u/1.60219E-13,1.60219E-16,1.66056E-27/
      DATA elec,Uion,MWelec/1.602E-19,13.6,9.109534E-31/ ![C,eV,kg/elec]
      DATA remnflux,rhoLiH,MWLiH/3.5e-8,.82e-3,1.332e-26/
C        [rem cm^2] for 2.5Mev neutrons, like from D(d,n)3He
C        [kg/cm^3]
C        MWLiH=[kg/molecule] molecular mass of lithium hydride shield
      DATA Wddn,Wddp,Wd3He,Wp11B/3.27,4.03,18.3,8.68/  ![MeV/rxn]
      DATA Wp6Li,Wdt,W23He,Wtt  /4.02,17.59,12.9,11.3/
      END
*****************************************************************************
```

A.2 comments.for

```
C   COMMENTS: MAIN VARIABLE DESCRIPTIONS (DEPENDENT AND INDEPENDENT)

C   program incrementable variables
C  count=generic counter
C  T=Ti+Te
C  Mr=ratio propellant mass flow to fuel mass flow
C  inc, increment for DO loop
C  expon  [unitless], number to which base is raised

C   functions
C  SVd3He=[cm^-3 s^-1] <ev> parameter for D-3He
C  SVddn=[cm^-3 s^-1] <ev> parameter for D(d,n)3He
C  SVddp=[cm^-3 s^-1] <ev> parameter for D(d,p)T

C   constants
C  mu0=1.2566E-6;         [H/m] permeability of free space
C  k=1.38066E-23;         [J/K] Boltzman's constant
C  J_keV=1.60219E-16;     [J/keV]
C  u=1.66056E-27;         [kg] atomic mass unit
C  g = 9.8062;            [m/s^2] gravitational constant
C  pi=3.141592654;           c
C  elec=1.602E-19            [C] electron charge
C  MWelec=[kg] electron mass
C  rhoLiH=[kg/cm^3] density of lithium hydride (shield material)
C  MWLiH=[kg] molecular mass of lithium hydride shield
C  remnflux=[rem cm^2] for 2.5MeV neutrons, like from D(d,n)3He (w2.45MeV)
C  Wd3He=[J] energy produced per D(3He,p)' fusion reaction
C  Wddn=[J] energy produced per D(d,n)3He fusion reaction
C  Wddp=[J] energy produced per D(d,p)T fusion reaction
C  Uion=[eV],ionization potential (13.6 for hydrogen)


C  INDEPENDENT VARIABLES

C   reactor
C  reactype= type of reactor (DPF,FRC, etc)
C  lower [keV] lower temperature boundry for DO loop
C  upper [keV] upper temperature boundry for DO loop
C  howmany=number of iterations to perform
C  Zj [unitless],atomic number of element j
C  Zk [unitless],atomic number of element k
C  Aj [unitless], atomic mass of element j
C  Ak [unitless], atomic mass of element k
C  kj [unitless],fractional density of element j
C  kk [unitless],fractional density of element k
C  rt [unitless], ratio T(electron)/T(ion)
C  Kc [unitless],fraction of cyclotron radiation absorbed in walls
C  cycabs [unitless],fraction of cyclotron radiation absorbed in plasma
```

```
C y=fraction of charged particle power retained in plasma
C Xn=ratio of propellant density to fuel density in reaction chamber
C etaGen=generator efficiency [Wth -> We]
C r1=[cm] inner shield radius (4c type shield)
C rpssngr=[cm] passenger distance from neutron source
C Xcone=fraction of sphere conical shield occupies
C specRad[kg/Wth]= specific mass of radiators
C specGen[kg/Wel]= specific mass of generators
C specExt[kg/Wel]= specific mass of external electrical eng. supply
C resmat=[jm] resitivity of theta-pinch material
C rhomat=[kg/cm^3] density of theta-pinch material
C tensmax=[Pa] tensile strength of theta-pinch material
C Xtens=ratio of theta pinch material tensile strength/stress (safety factor)
C maxdose=[rem] maximum dose allowable for person from reactor
C ddnspin=suppressing/enhancing factor from spinpolarization (1=no effect)
C ddpspin=suppressing/enhancing factor from spinpolarization
C d3Hespin=suppressing/enhancing factor from spinpolarization
C etaInj=injector efficiency [We -> Wplasma]
C specInj[kg/Wmdfuel]= specific mass of fusion fuel injectors

C  dpf
C PNCHRAD=[cm] radius of pinch
C RANODE=ANODE RADIUS  (cM)
C RCATH=CATHODE RADIUS  (cM)
C LANODE=ANODE LENGTH (cM)
C PNCHRAD=RADIUS OF PINCH  (cM)
C LPNCH =PINCH LENGTH (cM)
C FSNPLW=SNOWPLOW EFFICIENCY FACTOR, FRACTION OF INITIAL FILL GAS
C   WHICH IS ENTRAINED IN THE RUNDOWN.
C FPNCH =PINCH EFFICIENCY FACTOR, FRACTION OF GAS IN RUNDOWN WHICH
C   IS TRAPPED INSIDE THE PINCH.
C RHOION=INITIAL FILL GAS DENSITY  (KG/cm**3)
C VOLT=CHARGING POTENTIAL OF CAPACITOR BANKS  (V)
C CAP=INITIAL EXTERNAL CAPACITANCE (CAPACITOR BANK)  (F)
C LINIT=INITIAL INDUCTANCE OF EXTERNAL CIRCUIT  (H)
C ITERS=NUMBER OF ITERATIONS PERFORMED DURING EACH PINCH
C REPRATE=NUMBER OF FIRINGS PER UNIT TIME  (S**-1)
C PNCHTIM=DURATION OF STABLE PINCH PHASE  (S)
C DSCHRG=TIME FOR FILL GAS TO BE DISCHARGED  (S)
C IMAGNET=CURRENT NECESSARY TO PRODUCE 2 TESLA MAGNETIC FIELD  (A)
C specCap=SPECIFIC ENERGY OF CAPACITOR BANKS  (J/KG)
C volmix=[cm^3] volume of mixing chamber (for DPF only)
C   volmix=volcham for FRC
C Bnoz=[T] nozzle magnetic field

C  frc
C choice=particle confinement scaling law choice (Krall, VSLS, TRX, etc)
C dzplug=[cm] thickness of mirror opposing nozzle-end mirror.
C mult=fraction of gN(VSLS) achievable
C rate=parameter for finding gN for VSLS theory
C near=parameter for finding gN for VSLS theory
```

```
C  Lcham=[cm] chamber length
C  ls [cm], separatrix length
C  xs [unitless],ratio separatrix radius/coil radius
C  rcoil [cm], radius of FRC coil
C  XBplug=ratio plug magtetic field strength to theta pinch fld strength
C  XBnoz=ratio nozzle magtetic field strength to theta pinch fld strength
C  Xrplug=ratio inner radius of plug to coil radius
C  Xrnoz=ratio inner radius of nozzle to coil radius

C   rocket
C  Zprop=[unitless],atomic number of propellant
C  Aprop=[unitless], atomic mass of propellant
C  h0=[J/kg] initial enthalpy of cold propellant
C  hMat=[J/kg] enthalpy of propellant at high temperature (materials limited)
C  Xtemp=Tmix/Tthroat p Xtemp w 1.35 i.e. is nozzle characteristic
C  Xvel=VEXIT/VTHROAT p Xvel w 2.0    i.e. is nozzle characteristic
C  tmd=[days] mission time
C  payload=[kg] payload mass
C  shelter=[kg] mass of Solar Particle Event (SPE) and storm shelter
C  Ppayld=[W] power demands of payload
C  dznoz=[cm] thickness of nozzle coil

C   output
C  o0  }
C  o1  }
C  o2  } I/O toggles to specify what to write to file
C  .   }
C  .   }
C  o10 }


C       for dpf, n is dependent
C       Bnoz is dependent for FRC
C       volmix is dep. for FRC


C   DEPENDENT VARIABLES (passed to subroutines)

C   plasma parameters
C  Ti   [keV], average ion temperature
C  Te   [keV], average electron temperature
C  volplas=[cm^3] plasma volume
C  MWfuel=[kg] molecular mass of average fuel particle
C  fz [unitless], =kj*Zj+kk*Zk, describes number of electrons in plasma
C  n [cm^-3],total ion density in plasma
C  mdfuel=[kg/s] fuel mass flow rate
C  beta [unitless], kinetic plasma press/external magnetic field press
C  Bint=[T] internal magnetic field strength
C  Bext=[T] external magnetic field strength
C  RRd3He=[cm^-3 s^-1] reaction rate of D(3He,p)'
C  RRddn=[cm^-3 s^-1] reaction rate of D(d,n)3He
```

```
C  RRddp=[cm^-3 s^-1] reaction rate of D(d,p)T
C  tauN=[s] confinement time of particles in FRC plasma

C   frc parameters (dependent)
C  kappa=parameter for finding gN for VSLS theory
C  drcoil=[cm] theta pinch coil thickness
C  rplug=[cm] plug inner radius
C  drplug=[cm] plug thickness
C  Bplug=[T] plug magnetic field strength

C   dpf parameters (dependent)
C  IMAXSQ=MAXIMUM CURRENT SQUARED (AMPS)
C  IMAX=sqrt MAXIMUM CURRENT SQUARED (AMPS)
C  IMOPT=ASSUMED VALUE OF MAXIMUM ATTAINABLE CURRENT  (AMPS)
C  TOTEFF=TOTAL EFFICIENCY=FSNPLW*FPNCH
C  XSECTAR=CROSS SECTIONAL AREA OF FOCUS DEVICE  (cM**2)
C  NFTHRST=THRUST DUE TO EXPELLED (NON-PINCH) GASES  (N)
C  ELECTEN=TOTAL ELECTRICAL ENERGY TO BE SUPPLIED BY CAPACITORS  (W)

C   powers
C  Preactor=[W] necessary power to drive reactor
C  Pinj=[W] power injected into plasma (if nec. to maintain steady state)
C  Pbr  [W], bremstrahhlung radiation
C  Pcy  [W], cyclotron radiation
C  Pradwall=[W] thermal radiation power absorbed (to be dissipated)
C  Pth=[W] total thermal power to be dissipated
C  PelReq=[W] electrical power required
C  PelGen=[W] power produced by generators
C  PradRfl=[W] cyclotron rad reflected (abs. by prop and plasma)
C  Pradplas=[W] power absorbed by plasma from PradRfl
C  Ppayload=[W] power requirements of payload
C  Pcoil=[W] power necessary to produce Bcoil (also nec. to dissipate I^2yR)
C  Pprop=[W] power of heated propellant flow.
C  Pmix=[W] thermal power at mixnation point
C  Plp=[W] power contained in particle lost from plasma
C  PextReq=[W] external electrical power required
C  Pnoz=[W] power necessary to produce Bnoz (also nec. to dissipate I^2yR)
C  Pplug=[W] power necessary to produce Bplug (also nec. to dissipate I^2yR)
C  TFP=TOTAL FUSION POWER  (W)
C  w=ratio thermal radiation absorbed by propellant/total thermal rad.
C  gamGen=fraction of heated propellant sent to generators
C  gamCool=fraction of cold propellant sent to absorb thermal rad.

C   masses
C  reactor=[kg] reactor mass
C  injector=[kg] mass of fuel injector
C  powersys=[kg] mass of power system
C  MCAP=TOTAL CAPACITOR BANK MASS  (KG)
C  prop=[kg] total propellant mass
C  fuel=[kg] total fuel mass
C  nozzle=[kg] nozzle mass
```

```
C  radiator=[kg] radiator mass
C  shield=[kg] shield mass
C  mf=[kg] final system mass
C  m0=[kg] initial system mass = mf+prop+fuel

C   rocket parameters (dependent)
C  rnoz=[cm] magnetic nozzle inner radius
C  drnoz=[cm] magnetic nozzle thickness
C  Tmix=[K] nozzle mixnation temperature
C  n_mix=[cm^-3] density of particles at mixnation (for nozzle)
C  thrust=[N] total thrust produced
C  Isp=[s] specific impulse
C  tms=[sec] mission time
C  MWprop=[kg] molecular mass of propellant particle
C  mdprop=[kg/s] propellant mass flow rate
C  mdout=[kg/s] total mass flow rate out
C  ionized=fraction of ionized plasma
C  Bnoz=[T] nozzle magnetic strength

C   component
C  surf=ratio surface area avail. cyclo. rad. /total chamber surface area
C  volcham=[cm^3] reaction chamber volume
C  r1=[cm] distance of shield from reactor
```

A.3 comblk.for

```
      IMPLICIT REAL (a - z)
      CHARACTER choice*1,IncVar*8,out_name*12,pause*1
      LOGICAL o0,o1,o2,o3,o4,o5,o6,o7,o8,o9,o10,o11,o12,o13,o14,o15
      DOUBLE PRECISION ionized


C     common blocks shared by subroutines

C   program incrementable variables
      COMMON/PROG/count,IncVar,howmany,lower,upper,inc,operate,bomb,
     &loglin
C   constants (from DATA blocks)
      COMMON/CONST/mu0,k,g,pi,J_MeV,J_keV,u,elec,Uion,MWelec,remnflux,
     &rhoLiH,MWLiH,Wddn,Wddp,Wd3He,Wp11B,Wp6Li,Wdt,W23He,Wtt
C   independent variables (read in from .hep files)
      COMMON/INDEP/o0,o1,o2,o3,o4,o5,o6,o7,o8,o9,o10,o11,o12,o13,o14,
     &o15,choice,mult,rate,near,Lcham,ls,rcoil,xs,XBplug,XBnoz,Xrplug,
     &dzplug,RANODE,RCATH,LANODE,RHOION,VOLT,CAP,LINIT,FSNPLW,FPNCH,
     &LPNCH,PNCHRAD,REPRATE,PNCHTIM,ITERS,DSCHRG,IMAGNET,specCap,
     &volmix,reactype,rt,Kc,cycabs,y,Xn,etaGen,r1,rpssngr,Xcone,
     &specRad,specGen,specExt,resmat,rhomat,tensmax,Xtens,maxdose,
     &ddnspin,ddpspin,d3Hespin,specInj,etaInj,Zprop,Aprop,h0,hMat,
     &Xtemp,Xvel,Ppayld,dznoz,Xrnoz,payload,shelter,Xtank,T,deltav,
     &xprot,xdeut,xtrit,x3He,x6Li,x11B,xalfa
C   dependent variables (passed to subroutines)
      COMMON/DEPEN/Ti,Te,volplas,MWfuel,fz,n,mdfuel,beta,Bext,Bint,
     &RRddn,kappa,drcoil,rplug,drplug,Bplug,tauN,
     &TOTEFF,XSECTAR,NFTHRST,IMAX,ELECTEN,PjetNF,
     &Preactor,Pinj,Pbr,Pcy,PradRfl,Pradplas,Pradwall,Pth,Plp,Pmix,
     &PelGen,PelReq,PextReq,Pnoz,Pcoil,Pplug,TFP,w,gamCool,gamGen,
     &injector,Mcap,reactor,radiator,shield,nozzle,powersys,mf,m0,
     &prop,fuel,rnoz,drnoz,Bnoz,Tmix,n_mix,thrust,Isp,ionized,
     &tms,MWprop,mdprop,mdout,surf,volcham,tanks


C     for iterations tracking
91    FORMAT (A,F5.0,A)
92    FORMAT (A,F5.0,A,A,A,1PE11.3)

C   NO TEXT FORMAT
C     three places after decimal
131   FORMAT (1PE11.3)
132   FORMAT (1PE11.3,1PE11.3)
133   FORMAT (1PE11.3,1PE11.3,1PE11.3)
134   FORMAT (1PE11.3,1PE11.3,1PE11.3,1PE11.3)
135   FORMAT (1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3)
136   FORMAT (1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3)
137   FORMAT (1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3)
138   FORMAT (1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,
```

```
      &1PE11.3)
139    FORMAT (1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,
      &1PE11.3,1PE11.3)
130    FORMAT (1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,1PE11.3,
      &1PE11.3,1PE11.3,1PE11.3)

C     TEXT FORMAT
191    FORMAT (A,1PE11.3)
192    FORMAT (A,1PE11.3,A,1PE11.3)
193    FORMAT (A,1PE11.3,A,1PE11.3,A,1PE11.3)
```

A.4 getvar.for

```
C23456789                                                          0
      SUBROUTINE getvar

      INCLUDE comblk.for

*****     INPUTS     *****
C   read in data from files
C   45 reactor
      OPEN (UNIT=45, FILE='REACTOR.hep', STATUS='OLD')
      READ(45,*)
      READ(45,*)reactype
      READ(45,*)
      READ(45,'(A8)')IncVar
      READ(45,*)
      READ(45,*)howmany
      READ(45,*)
      READ(45,*)lower
      READ(45,*)
      READ(45,*)upper
      READ(45,*)
      READ(45,*)loglin
      READ(45,*)
      READ(45,*)T
      READ(45,*)
      READ(45,*)xprot
      READ(45,*)
      READ(45,*)xdeut
      READ(45,*)
      READ(45,*)xtrit
      READ(45,*)
      READ(45,*)x3He
      READ(45,*)
      READ(45,*)x6Li
      READ(45,*)
      READ(45,*)x11B
      READ(45,*)
      READ(45,*)xalfa
      READ(45,*)
      READ(45,*)rt
      READ(45,*)
      READ(45,*)Kc
      READ(45,*)
      READ(45,*)cycabs
      READ(45,*)
      READ(45,*)y
      READ(45,*)
      READ(45,*)Xn
      READ(45,*)
      READ(45,*)etaGen
```

```
      READ(45,*)
      READ(45,*)r1
      READ(45,*)
      READ(45,*)rpssngr
      READ(45,*)
      READ(45,*)Xcone
      READ(45,*)
      READ(45,*)specRad
      READ(45,*)
      READ(45,*)specGen
      READ(45,*)
      READ(45,*)specExt
      READ(45,*)
      READ(45,*)resmat
      READ(45,*)
      READ(45,*)rhomat
      READ(45,*)
      READ(45,*)tensmax
      READ(45,*)
      READ(45,*)Xtens
      READ(45,*)
      READ(45,*)maxdose
      READ(45,*)
      READ(45,*)ddnspin
      READ(45,*)
      READ(45,*)ddpspin
      READ(45,*)
      READ(45,*)d3Hespin
      CLOSE (UNIT=45)

C   41 frc
      IF(reactype.EQ.3)THEN
      OPEN (UNIT=41, FILE='FRC.hep', STATUS='OLD')
      READ(41,*)
      READ(41,'(A1)')choice
      READ(41,*)
      READ(41,*)mult
      READ(41,*)
      READ(41,*)rate
      READ(41,*)
      READ(41,*)near
      READ(41,*)
      READ(41,*)Lcham
      READ(41,*)
      READ(41,*)ls
      READ(41,*)
      READ(41,*)rcoil
      READ(41,*)
      READ(41,*)xs
      READ(41,*)
      READ(41,*)n
```

```
      READ(41,*)
      READ(41,*)XBplug
      READ(41,*)
      READ(41,*)XBnoz
      READ(41,*)
      READ(41,*)Xrplug
      READ(41,*)
      READ(41,*)dzplug
      READ(41,*)
      READ(41,*)Xrnoz
      READ(41,*)
      READ(41,*)specInj
      READ(41,*)
      READ(41,*)etaInj
      CLOSE (UNIT=41)
      ENDIF

C   43 dpf
      IF(reactype.EQ.2)THEN
      OPEN (UNIT=43,FILE='DPF.hep',STATUS='OLD')
      READ(43,*)
      READ(43,*)RANODE
      READ(43,*)
      READ(43,*)RCATH
      READ(43,*)
C       kr
C       no lpnch
      READ(43,*)LANODE
      READ(43,*)
      READ(43,*)RHOION
      READ(43,*)
      READ(43,*)VOLT
      READ(43,*)
      READ(43,*)CAP
      READ(43,*)
      READ(43,*)LINIT
      READ(43,*)
      READ(43,*)FSNPLW
      READ(43,*)
      READ(43,*)FPNCH
      READ(43,*)
      READ(43,*)LPNCH
      READ(43,*)
      READ(43,*)PNCHRAD
      READ(43,*)
      READ(43,*)REPRATE
      READ(43,*)
      READ(43,*)PNCHTIM
      READ(43,*)
      READ(43,*)ITERS
      READ(43,*)
```

```
      READ(43,*)DSCHRG
      READ(43,*)
      READ(43,*)IMAGNET
      READ(43,*)
      READ(43,*)Bnoz
      READ(43,*)
      READ(43,*)specCap
      READ(43,*)
      READ(43,*)Xrnoz
      READ(43,*)
      READ(43,*)specInj
      READ(43,*)
      READ(43,*)etaInj
      CLOSE (UNIT=43)
      ENDIF

C   47 rocket
      OPEN (UNIT=47, FILE='ROCKET.hep', STATUS='OLD')
      READ(47,*)
      READ(47,*)Mr
      READ(47,*)
      READ(47,*)Zprop
      READ(47,*)
      READ(47,*)Aprop
      READ(47,*)
      READ(47,*)h0
      READ(47,*)
      READ(47,*)hMat
      READ(47,*)
      READ(47,*)Xtemp
      READ(47,*)
      READ(47,*)Xvel
      READ(47,*)
      READ(47,*)deltav
      READ(47,*)
      READ(47,*)payload
      READ(47,*)
      READ(47,*)Ppayld
      READ(47,*)
      READ(47,*)shelter
      READ(47,*)
      READ(47,*)dznoz
      READ(47,*)
      READ(47,*)volmix
      READ(47,*)
      READ(47,*)Xtank
      READ(47,*)
      READ(47,*)Tmix
      CLOSE (UNIT=47)

C   49 output
```

```
      OPEN (UNIT=49, FILE='OUTPUT.hep', STATUS='OLD')
      READ(49,*)
      READ(49,*)o0
      READ(49,*)
      READ(49,*)o1
      READ(49,*)
      READ(49,*)o2
      READ(49,*)
      READ(49,*)o3
      READ(49,*)
      READ(49,*)o4
      READ(49,*)
      READ(49,*)o5
      READ(49,*)
      READ(49,*)o6
      READ(49,*)
      READ(49,*)o7
      READ(49,*)
      READ(49,*)o8
      READ(49,*)
      READ(49,*)o9
      READ(49,*)
      READ(49,*)o10
      READ(49,*)
      READ(49,*)o11
      READ(49,*)
      READ(49,*)o12
      READ(49,*)
      READ(49,*)o13
      READ(49,*)
      READ(49,*)o14
      READ(49,*)
      READ(49,*)o15
      CLOSE (UNIT=49)

*****     DISPLAY INPUTS     *****
C     display input values to check file read was accurate
      IF(o0)THEN
      WRITE(*,*)'Press <Enter> to continue'
      READ(*,'(A1)')pause

C   45 reactor
        WRITE(*,*)'Data from file: REACTOR'
        WRITE(*,*)'Incrementing variable: ',IncVar
        WRITE(*,191)' howmany =',howmany
        WRITE(*,191)' lower =',lower
        WRITE(*,191)' upper =',upper
        WRITE(*,191)' T [keV] =',T
        WRITE(*,191)' xprot =',xprot
        WRITE(*,191)' xdeut =',xdeut
```

```
      WRITE(*,191)' xtrit =',xtrit
      WRITE(*,191)' x3He  =',x3He
      WRITE(*,191)' x6Li  =',x6Li
      WRITE(*,191)' x11B  =',x11B
      WRITE(*,191)' xalfa =',xalfa
      WRITE(*,191)' rt =',rt
      WRITE(*,191)' Kc     =',Kc
      WRITE(*,191)' cycabs =',cycabs
      WRITE(*,191)' y      =',y
      WRITE(*,191)' Xn =',Xn
      WRITE(*,191)' etaGen =',etaGen
      WRITE(*,191)' r1[cm]      =',r1
      WRITE(*,191)' rpssngr[cm] =',rpssngr
      WRITE(*,191)' Xcone =',Xcone
      WRITE(*,191)' specRad[kg/kWth]     =',specRad
      WRITE(*,191)' specGen[kg/kWel]     =',specGen
      WRITE(*,191)' specExt[kg/kWel]     =',specExt
      WRITE(*,*)'Press <Enter> to continue'
      READ(*,'(A1)')pause
      WRITE(*,*)'Data from file: REACTOR (continued)'
      WRITE(*,191)' resmat[jm]=',resmat
      WRITE(*,191)' rhomat[kg/cm^3]=',rhomat
      WRITE(*,191)' tensmax[Pa]=',tensmax
      WRITE(*,191)' Xtens    =',Xtens
      WRITE(*,191)' maxdose[rem] =',maxdose
      WRITE(*,191)' ddnspin  =',ddnspin
      WRITE(*,191)' ddpspin  =',ddpspin
      WRITE(*,191)' d3Hespin =',d3Hespin
      WRITE(*,*)'Press <Enter> to continue'
      READ(*,'(A1)')pause

C   41 frc
     IF(reactype.EQ.3)THEN
      WRITE(*,*)'Data from file: FRC'
      WRITE(*,*)'choice =',choice
      WRITE(*,191)' mult =',mult
      WRITE(*,191)' rate =',rate
      WRITE(*,191)' near =',near
      WRITE(*,191)' Lcham[cm] =',Lcham
      WRITE(*,191)' ls   [cm] =',ls
      WRITE(*,191)' rcoil[cm] =',rcoil
      WRITE(*,191)' xs =',xs
      WRITE(*,191)' n[cm^-3] =',n
      WRITE(*,191)' XBplug   =',XBplug
      WRITE(*,191)' XBnoz    =',XBnoz
      WRITE(*,191)' Xrplug   =',Xrplug
      WRITE(*,191)' dzplug[cm]=',dzplug
      WRITE(*,191)' Xrnoz    =',Xrnoz
      WRITE(*,191)' specInj[kg/kWmdfuel] =',specInj
      WRITE(*,191)' etaInj =',etaInj
      WRITE(*,*)'Press <Enter> to continue'
```

```
      READ(*,'(A1)')pause
    ENDIF

C   43 dpf
    IF(reactype.EQ.2)THEN
      WRITE(*,*)'Data from file: DPF'
      WRITE(*,191)' RANODE [cm]=',RANODE
      WRITE(*,191)' RCATH  [cm]=',RCATH
      WRITE(*,191)' LANODE [cm]=',LANODE
      WRITE(*,191)' RHOION =',RHOION
      WRITE(*,191)' VOLT [V]=',VOLT
      WRITE(*,191)' CAP   [F]=',CAP
      WRITE(*,191)' LINIT [H]=',LINIT
      WRITE(*,191)' FSNPLW =',FSNPLW
      WRITE(*,191)' FPNCH  =',FPNCH
      WRITE(*,191)' LPNCH    [cm]=',LPNCH
      WRITE(*,191)' PNCHRAD [cm]=',PNCHRAD
      WRITE(*,191)' REPRATE [s^-1]=',REPRATE
      WRITE(*,191)' PNCHTIM [s]=',PNCHTIM
      WRITE(*,191)' ITERS =',ITERS
      WRITE(*,191)' DSCHRG [s]=',DSCHRG
      WRITE(*,191)' IMAGNET =',IMAGNET
      WRITE(*,*)'Press <Enter> to continue'
      READ(*,'(A1)')pause
      WRITE(*,*)'Data from file: DPF  (continued)'
      WRITE(*,191)' Bnoz [T]=',Bnoz
      WRITE(*,191)' specCap [Kj/kg]=',specCap
      WRITE(*,191)' Xrnoz =',Xrnoz
      WRITE(*,191)' specInj =',specInj
      WRITE(*,191)' etaInj  =',etaInj
      WRITE(*,*)'Press <Enter> to continue'
      READ(*,'(A1)')pause
    ENDIF

C   47 rocket
      WRITE(*,*)'Data from file: ROCKET'
      WRITE(*,191)' Mr =',Mr
      WRITE(*,191)' Zprop =',Zprop
      WRITE(*,191)' Aprop =',Aprop
      WRITE(*,191)' h0  [J/kg] =',h0
      WRITE(*,191)' hMat[J/kg] =',hMat
      WRITE(*,191)' Xtemp =',Xtemp
      WRITE(*,191)' Xvel =',Xvel
      WRITE(*,191)' deltav[m/s] =',deltav
      WRITE(*,191)' payload[kg] =',payload
      WRITE(*,191)' Ppayld[W] =',Ppayld
      WRITE(*,191)' shelter[kg] =',shelter
      WRITE(*,191)' dznoz [cm]=',dznoz
      WRITE(*,191)' volmix [cm^3]=',volmix
      WRITE(*,191)' Xtank =',Xtank
      WRITE(*,191)' Tmix[K] (initial guess)=',Tmix
```

```
      WRITE(*,*)'Press <Enter> to continue'
      READ(*,'(A1)')pause

    ENDIF
    END
C     end subroutine getvar
```

A.5 preloop.for

```
C23456789                                                              0
      SUBROUTINE preloop

      INCLUDE comblk.for

*****     BEGIN GENERIC PRE-LOOP(EXT) CALCULATIONS     *****

      mui=xprot*1.+xdeut*2.+xtrit*3.+x3He*3.+x6Li*6.+x11B*11.+xalfa*4.
C       [unitless] average atomic mass
      MWfuel=u*mui
C       [kg] ave mass of fuel ion

      fz=xprot+xdeut+xtrit+x3He*2.+x6Li*3.+x11B*5.+xalfa*2.
C     fz=(kj*Zj+kk*Zk+...)
C       [unitless] average atomic no of particles

      MWprop=u*(Aprop)
C       [kg] mass of propellant ion
*****     END GENERIC PRE-LOOP(EXT) CALCULATIONS     *****
      WRITE(*,*) 'Name of file to output? (8+3 char. max) '
      READ(*,'(A12)')out_name
      OPEN (UNIT=55, FILE=out_name, STATUS='NEW')
      WRITE(*,*)'Writing to file ',out_name


      IF(reactype.EQ.2)THEN
        WRITE(55,*)'DPF'
*****     BEGIN DPF PRE-LOOP CALCULATIONS     *****
C   CALCULATE FOCUS PARAMETERS
      lpnch = rcath - ranode
      pnchrad = ranode/kr

      XSECTAR=pi*(RCATH**2. - RANODE**2.)
C       [cm^2]
      volcham=lanode*XSECTAR
C       [cm^3]
      volplas=pi*PNCHRAD**2.*LPNCH
C       [cm^3]
      TOTEFF=FSNPLW * FPNCH

      surf=(2.*pi*RCATH*LANODE + 2.*pi*RANODE*LANODE +   pi*RCATH**2.)
     &/    (2.*pi*RCATH*LANODE + 2.*pi*RANODE*LANODE + 2.*pi*RCATH**2.)
C       fraction of surface area not "holes", ie ends

*****     END DPF PRE-LOOP CALCULATIONS     *****

      ELSEIF(reactype.EQ.3)THEN
        WRITE(55,*)'FRC'
*****     BEGIN FRC PRE-LOOP(EXT) CALCULATIONS     *****
```

```
      beta=1-xs**2./2.
C       FRC formula
      volplas=ls*pi*(xs*rcoil)**2.
C       cm^3
      volcham=Lcham*pi*rcoil**2.
C       cm^3
C   compute particle confinement time using one of three scaling methods.
      WRITE(*,*)'Particle Confinement Scaling: '
      IF (choice.EQ.'V'.OR.choice.EQ.'v') THEN
C     VSLS Theory
        WRITE(55,*)'VSLS Theory'
        WRITE(*,*)'VSLS Theory'
        kappa=BETA
10      IF ((TANH(kappa)/kappa).GT.BETA) THEN
          kappa=kappa*RATE
        ELSE
          kappa=kappa/RATE
        ENDIF
        DIFF=ABS(BETA - TANH(kappa)/kappa)/beta
        IF (DIFF.GT.NEAR) GOTO 10
      ELSEIF (choice.EQ.'T'.OR.choice.EQ.'t') THEN
        WRITE(55,*)'TRX-2 Experiment'
        WRITE(*,*)'TRX-2 Experiment'
      ELSEIF (choice.EQ.'K'.OR.choice.EQ.'k') THEN
        WRITE(55,*)'Krall Theory'
        WRITE(*,*)'Krall Theory'
      ELSEIF (choice.EQ.'L'.OR.choice.EQ.'l') THEN
        WRITE(55,*)'Lower Hybrid Drift Theory'
        WRITE(*,*)'Lower Hybrid Drift Theory'
      ENDIF
*****     END FRC PRE-LOOP(EXT) CALCULATIONS     *****
      ENDIF

C   write header to table-format output file
      IF(o2.AND.o9)WRITE(55,*)'T, QppPfus/Pinj'
      IF(o3.AND.o9)WRITE(55,*)'T,mdout,prop,fuel'
      IF(o4.AND.o9)WRITE(55,*)'T,injector,radiator,shield,reactor,',
     &'nozzle,powersys,payload,shelter'
      IF(o5.AND.o9)WRITE(55,*)'T,Pradwall,Pcoil,Pnoz'
      IF(o6.AND.o9)WRITE(55,*)'T,Ppayld,Pinj/etaInj,Pcoil,Pplug,Pnoz'
      IF(o7.AND.o9)WRITE(55,*)'T,w,gamCool,gamGen,PelGen/PextReq'
      IF(o8.AND.o9)WRITE(55,*)'T, Thrust, Non-Fusion Thrust, ratio'

c      IF(o10.AND.o9)WRITE(55,*)
c     &'time,xprot,xdeut,xtrit,x3He,x6Li,x11B,xalfa,n'
c      IF(o10.AND.o9)WRITE(55,*)
c     &'time,PFddn,PFddp,PFd3He,PFp11B,PFp6Li,PFdt,PF23He,Pneut,PFTOT'
      IF(o10.AND.o9)WRITE(55,*)'T,TFP,TPneut,TFP/TPneut'

      IF(o12.AND.o9)WRITE(55,*)
     &'T, deltaV, tmd, payload/m0'
```

```
      IF(o13.AND.o9)WRITE(55,*)
     &'T, mdfuel*Vrun**2/2, TDELTAP, Pprop, Pmix, Pjet'
      IF(o14.AND.o9)WRITE(55,*)
     &'T, PjetNF/Pmix, Pc/Pmix, Pprop/Pmix, Pjet[W]'
      IF(o15.AND.o9)WRITE(55,*)
     &'T, Pjet, PjetNF, Pinj'

c      IF(o10)THEN
c        WRITE(*,*) 'VARITOP destination file name? (8+3 char. max) '
c        READ(*,'(A12)')out_name
c        OPEN (UNIT=65, FILE=out_name, STATUS='NEW')
c        WRITE(*,*)'Writing to file ',out_name
c      ENDIF


      END
C       subroutine preloop
```

A.6 gettau.for

```
      SUBROUTINE gettau
C     finds tauN

C     truly local variables
      REAL const,Fkrall,R,omegaci,rhoi,Scap,stau,Wtau,fs

      IMPLICIT REAL (a - z)
      CHARACTER choice*1,IncVar*8
      LOGICAL o0,o1,o2,o3,o4,o5,o6,o7,o8,o9,o10,o11,o12,o13,o14,o15
      DOUBLE PRECISION ionized

      INCLUDE comblk.for


********* FRC PARTICLE CONFINEMENT SCALING tauN ***************
        IF (choice.EQ.'T'.OR.choice.EQ.'t') THEN
C       TRX-2 Experiment
          const=1.e-10
C         calibrates scaling law to give confinement times in proper units
          tauN=(xs*rcoil)**1.*(n/1.e15)**0.9*T**1.5*const
C           from L.Steinhauer personal conversations, Tusz. FRC review
C           n/1e15 converts n to [10^15 cm^-3] for scaling law
        ELSEIF (choice.EQ.'V'.OR.choice.EQ.'v') THEN
C       VSLS Theory
          const=6.e-6
C         calibrates scaling law to give confinement times in proper units
          tauN=(Ti*1.e3)**0.2*kappa**2.7*rcoil**1.8*Bext**2.4*
     &    (n/1.e15)**(-1.)*const*mult
C           from Miley Nuc.Instm.Meth.
C           Ti*1e3 converts keV to eV
C           n/1e15 converts n[cm^-3] to n[10^15 cm^-3] for scaling law
        ELSEIF (choice.EQ.'K'.OR.choice.EQ.'k') THEN
C       Krall Theory
          Fkrall=pi/(3.*(1.+rt)**0.5)
          tauN=3.e-8*(Bext*1.e4)*(xs*rcoil)**2./(Fkrall*(Te*1.e3)*
     &    (1.+rt)**0.5)
C           from Krall Phys.Fluids.
C           input tauN[s], Bext[T], xs*rcoil[cm], T[keV]
C           Ti*1e3 converts keV to eV
C           Bext*1e4 converts T to Gauss for Krall's scaling
        ELSEIF (choice.EQ.'L'.OR.choice.EQ.'l') THEN
C       Lower Hybrid Drift Theory
          R=(rcoil*xs/100.)/(2.**0.5)
          omegaci=elec*Bext/MWfuel
          rhoi=(J_keV*Ti/MWfuel)**0.5/omegaci
          Scap=R/rhoi
          stau=xs**2.*Scap/2.**(3./2.)*3./4.
C           corresponding to del0 from Phys Fluids 26(1983)1626 Hoffmann/Milroy
          Wtau=3.
```

```
C          between 2-4, but tauN not very sensitive to Wtau anyway
         fs=1.
C          "arbitrary" parameter
         tauN=0.13*fs**(-1.)*beta*(stau**3.)*Scap/(1.+Te/Ti)/Bint*
     &  (1.+Wtau/(stau/3.))**3./1.e6
C        1e6 converts tauNs[fs] to tauN[s]
C        from Nuc Fus 24(1984)1540 Slough et al. eq. 7 for spatially
C        varying (LHD) resistivity profile
        ENDIF
C     now have tauN

      END
C       real function tauN
```

A.7 sigmav.for

```
C  given ion temperature (keV), calculates sigma-v parameter
Coefficients thanks to: Larry T. Cox, AL(AFSC)/LSVF

C  for D(d,n)3He rxn;
      REAL FUNCTION SVddn(T)
      REAL a,b,c,d,T,x,LOG10
        x=LOG10(T)
        a=  0.2981119
        b= -2.082958
        c=  5.701349
        d=-22.08780
        SVddn=10**(a*x**3+b*x**2+c*x+d)
      END
C     function SVddn


C  for D(d,p)T rxn;
      REAL FUNCTION SVddp(T)
      REAL a,b,c,d,e,T,x,LOG10
        x=LOG10(T)
        a= -0.05730514
        b=  0.5716992
        c= -2.404634
        d=  5.643589
        e=-21.97105
        SVddp=10**(a*x**4+b*x**3+c*x**2+d*x+e)
      END
C     function SVddp


C  for 3He(d,p)' rxn;
      REAL FUNCTION SVd3He(T)
      REAL a,b,c,d,T,x,LOG10
        x=LOG10(T)
        a=  0.3535589
        b= -3.310354
        c= 10.10471
        d=-25.67344
        SVd3He=10**(a*x**3+b*x**2+c*x+d)
      END
C     function SVd3He

C  for 11B(p,')'+' rxn;
      REAL FUNCTION SVp11B(T)
      REAL a,b,c,d,T,x,LOG10
        x=LOG10(T)
        a=  0.6879438
        b= -5.833501
        c= 17.40498
```

```
      d=-33.51636
      SVp11B=10**(a*x**3+b*x**2+c*x+d)
     END
C    function SVp11B

C  for 6Li(p,3He)' rxn;
     REAL FUNCTION SVp6Li(T)
     REAL a,b,c,d,T,x,LOG10
      x=LOG10(T)
      a=  0.5712202
      b= -4.265839
      c= 11.87649
      d=-28.12494
      SVp6Li=10**(a*x**3+b*x**2+c*x+d)
     END
C    function SVp6Li

C  for T(d,n)' rxn;
     REAL FUNCTION SVdt(T)
     REAL a,b,c,d,T,x,LOG10
      x=LOG10(T)
      a=  0.2708006
      b= -2.421732
      c=  6.318869
      d=-20.15761
      SVdt=10**(a*x**3+b*x**2+c*x+d)
     END
C    function SVdt

C  for 3He(3He,2p)' rxn;
     REAL FUNCTION SV23He(T)
     REAL a,b,c,d,T,x,LOG10
      x=LOG10(T)
      a=  0.6392849
      b= -5.274763
      c= 16.43308
      d=-11.10835
      SV23He=10**(a*x**3+b*x**2+c*x+d-24)
c     -24 above is for barns
     END
C    function SV23He

C  for T(t,2n)' rxn;
     REAL FUNCTION SVtt(T)
     REAL a,b,c,d,e,T,x,LOG10
      x=LOG10(T)
      a= -0.07238949
      b=  0.6854508
      c= -2.827826
      d=  6.513504
      e=-22.72402
```

```
      SVtt=10**(a*x**4+b*x**3+c*x**2+d*x+e)
     END
C    function SVtt
```

A.8 pow-d.for

```
C23456789                                                      0
      SUBROUTINE pow_d

      INCLUDE comblk.for

*****     BEGIN DPF PLASMA POWER BALANCE     *****
C   PLASMA TEMPERATURE

      AU=(W*Idot0*Ldot*Ldot)**(1./3.)
      AI=(W*Idot0/Ldot)**(1./3.)
      At=(W/(Idot0*Idot0*Ldot))**(1./3.)

      Imax   = 0.64 * AI
      volt0  = 2.12 * AU
      voltmax= 0.64 * AU
      trise  = 1.45 * At

C   RUNDOWN VELOCITY
C   THE RUNDOWN VELOCITY IS CALCULATED USING THE MOMENTUM EQUATION
      Vrun=Ldot*2*pi/(mu0*log(rcath/ranode))  ![m/s]
      mdfuel=MWfuel*inven*REPRATE  ![kg/s]

C     DETERMINE INJECTED POWER BY RATE OF BANK ENERGIES DELIVERED
        Pinj=W*REPRATE ![W]

C  initial loop values
C   PINCH NUMBER DENSITY
      n=inven/volplas ![cm^-3]
C   define initial element number densities in pinch via specified fractions
      prot=n*xprot
      deut=n*xdeut
      trit=n*xtrit
      hel3=n*x3He
      lith=n*x6Li
      boron=n*x11B
      alfa=n*xalfa ![cm^-3]

      fz=xprot+xdeut+xtrit+x3He*2.+x6Li*3.+x11B*5.+xalfa*2.
      fzSQ=xprot+xdeut+xtrit+x3He*4.+x6Li*9.+x11B*25.+xalfa*4.

      TFP=0
      TPLOSS=0
      TDELTAP=0
      Pbr=0
      Pcy=0
      TPneut=0

      iprot =xprot
      ideut =xdeut
```

```
      itrit =xtrit
      i3He  =x3He
      i6Li  =x6Li
      i11B  =x11B
      ialfa =xalfa

C     FOR ONE PULSE
***** begin DPF internal loop *****
      DO 2002 count2=1,iters,1
        IF(MOD(100.*count2/iters,5.).EQ.0.)
     &  WRITE(*,91)'+  dpf',count2/iters*100.,'%'
C         to determine progress

        RRddn =deut*(deut *SVddn(T) )
        RRddp =deut*(deut *SVddp(T) )
        RRd3He=deut*(hel3 *SVd3He(T))
        RRp11B=prot*(boron*SVp11B(T))
        RRp6Li=prot*(lith *SVp6Li(T))
        RRdt  =deut*(trit *SVdt(T)  )
        RR23He=hel3*(hel3 *SV23He(T))
        RRtt  =trit*(trit *SVtt(T)  ) ! [cm^-3 s^-1]

        vrpi=volplas*REPRATE*PNCHTIM/ITERS

C     DETERMINE CHARGED FUSION POWER FROM PINCH
        PFddn =vrpi*J_MeV*RRddn*Wddn*0.25
        PFddp =vrpi*J_MeV*RRddp*Wddp
        PFd3He=vrpi*J_MeV*RRd3He*Wd3He
        PFp11B=vrpi*J_MeV*RRp11B*Wp11B
        PFp6Li=vrpi*J_MeV*RRp6Li*Wp6Li
        PFdt  =vrpi*J_MeV*RRdt*Wdt*0.20
        PF23He=vrpi*J_MeV*RR23He*W23He
        PFtt  =vrpi*J_MeV*RRtt*Wtt
        Pneut =vrpi*J_MeV*(RRddn*Wddn*0.75+RRdt*Wdt*0.80)
        PFTOT=PFddn+PFddp+PFd3He+PFp11B+PFp6Li+PFdt+PF23He+PFtt  ![W]

C       average atomic number of particles in pinch


        fz=iprot+ideut+itrit+i3He*2.+i6Li*3.+i11B*5.+ialfa*2.
        fzSQ=iprot+ideut+itrit+i3He*4.+i6Li*9.+i11B*25.+ialfa*4.

        PBREM=n*(n*5.35E-31)*fz*fzSQ*SQRT(Te)*vrpi
C         n[cm^-3]; Te[keV]; Pbrem,Pcyc [W];

        prot=prot+(-RRp11B-RRp6Li+RRddp+RRd3He+2.*RR23He)*PNCHTIM/ITERS
        deut=deut+(-RRdt-2.*RRddn-2.*RRddp-RRd3He)*PNCHTIM/ITERS
        trit=trit+(-RRdt-2.*RRtt+RRddp)*PNCHTIM/ITERS
        hel3=hel3+(-RRd3He-2.*RR23He+RRddn+RRp6Li)*PNCHTIM/ITERS
        lith=lith+(-RRp6Li)*PNCHTIM/ITERS
        boron=boron+(-RRp11B)*PNCHTIM/ITERS
```

```
         alfa=alfa+(+RRdt+RRtt+RRd3He+RRp6Li+3.*RRp11B+RR23He)
     &   *PNCHTIM/ITERS

         n=prot+deut+trit+hel3+lith+boron+alfa
         iprot =prot/n
         ideut =deut/n
         itrit =trit/n
         i3He  =hel3/n
         i6Li  =lith/n
         i11B  =boron/n
         ialfa =alfa/n

         TFP=TFP+PFTOT
         TPneut=TPneut+Pneut
         Pbr=Pbr+PBREM
         Pcy=Pcy+PCYC
C        [W]

2002     CONTINUE
***** end internal loop *****

         IF(o10)WRITE(55,134)T,TFP,TPneut,TFP/TPneut

         TDELTAP=TFP-Pbr
         Plp=TDELTAP
C          [W]

*****      END DPF PLASMA POWER BALANCE      *****

       END
C        subroutine pow_d
```

A.9 therm-d.for

```
C23456789                                                     0
      SUBROUTINE therm_d

      INCLUDE comblk.for

*****     BEGIN DPF THERMAL POWER      *****
C     CYCLOTRON RADIATION ABSORBED IN WALL AND ELECTRODES
        Pradwall=Pbr+Kc*Pcy
C       assumes walls are reflective to Bremsstrahlung radiation

C     TOTAL POWER DISSIPATED IN ELECTRODES BY OHMIC HEATING
        Pelctrd=Imax**2.*resmat*(LANODE/100.)*(1./(pi*(
     &RANODE/100.)**2.)
     &+1./(pi*((RCATH/100.)**2.-(RANODE/100.)**2.)))*DSCHRG*REPRATE
C     TOTAL POWER TO BE REMOVED FROM THE WALLS AND ELECTRODES AND MAGNET


c       on first run, Tmix is not yet calculated: need nozzle thermal
c       thermal power to determine Tmix, and Tmix to determine
c       nozzle thermal power

c       check Tmix, ionization; if out of bounds, say so; if need
c       conventional nozzle, Bnoz = 0, drnoz fixed at XX
c       drnoz minimum = 1 cm

C       nozzle strength Bnoz depends on mixnation temperature and ion
C       density of effluent
C       Bnoz=10.**(0.5*LOG10(Tmix[eV])+LOG10(n_mix)-17.05)
C       from "Char. Flow Thru Mag. Noz", Sgro, Glasser, et al.

C       n_ions=n_fuel + *n_prop
        n_ions=(n*volplas+Xn*n*(volcham-volplas))/volcham
C         [cm^-3]
C       n_elec=n_fuel*Z_fuel + n_prop*Z_prop
        n_elec=(n*volplas*fz+Xn*n*(volcham-volplas)*Zprop)/volcham
C         [cm^-3]
        n_mix=(n_ions+n_elec)*volplas/volmix

      Bnoz=10.**(0.5*LOG10(Tmix/11604.85)+LOG10(n_mix)-17.05)

C       calculate DPF magnetic nozzle strength for file input.
C       don't calculate above because read in Bnoz (Tmix,n_mix
C       depend on Pnoz, Bnoz i.e. not calculated yet
C       ultimately only affects mass

C    USE SAHA EQUATION TO CALCULATE DEGREE OF PROPELLANT IONIZATION
C    FOR MAGNETIC NOZZLE (assumes hydrogen propellant)
C    if ionization isn't near 100% mag. nozzle is not as effective
```

```
      IF (Tmix.LT.2500) THEN
C        use conventional nozzle
         drnoz=10.
C          [cm]
         Pnoz=0.
C          [W]
      ELSE
         CALL saha (Uion,Tmix,n_mix,ionized)
         IF (ionized.LT.0.3) THEN
c             WRITE(*,*)' WAS_D: *Not ionized ',
c     &'enough for magnetic nozzle*'
            drnoz=20.
C             [cm]
            Pnoz=0.
C             [W]
         ELSE
            rnoz=Xrnoz*rcath
            drnoz=rnoz/(tensmax/Xtens*2.*mu0/Bnoz**2.-1./3.)
            IF (drnoz.LT.1) drnoz=1
            alphg=(rnoz+drnoz)/rnoz
            betag=dznoz/(2*rnoz)
            gab=(betag/(2.*pi*(alphg**2.-1.)))**0.5*
     &LOG((alphg+(alphg**2.+betag**2.)**0.5)/(1.+(1.+betag**2.)**0.5))
            lambda=0.9
            Pnoz=(Bnoz/(lambda*mu0*gab))**2.*resmat*rnoz/100.
C             rnoz/100. converts rnoz[cm] to rnoz[m]
         ENDIF
      ENDIF

      Preactor=Pelctrd
      Pth=Pradwall+Preactor+Pnoz

*****    END DPF THERMAL POWER     *****

      END
C       subroutine was_d
```

A.10 pow-f.for

```
C23456789                                                          0
      SUBROUTINE pow_f

      COMMON/...

*****     BEGIN FRC PLASMA POWER BALANCE     *****

C     plasma pressure balance: Press_mag,ext=Press_plas + Press_mag,int
C     and def <a>pPress_plas/Press_mag,ext=1.-xs**2./2.
        Bext=(n*1.e6*J_keV*T/(beta/(2.*mu0)))**0.5
C         n*1.e6 converts n[cm^-3] to n[m^-3]
        Bint=((Bext**2./(2.*mu0)-n*1.e6*J_keV*T)*2.*mu0)**0.5

C     find particle confinement time
C     tauN(ls,xs,rcoil,n,const,rs=xs*rcoil,B,Te,Ti,kappa,etc...)
        CALL gettau

C     D(3He,p)'  D(d,n)3He  D(d,p)T
        RRddn=ddnspin*(Kj*n)**2./2.*SVddn(T)*volplas
        RRddp=ddpspin*(Kj*n)**2./2.*SVddp(T)*volplas
        RRd3He=d3Hespin*kj*kk*n**2.*SVd3He(T)*volplas
C         [cm^-3 s^-1]
        Pc=RRd3He*Wd3He+RRddp*Wddp+RRddn*Wddn/4.
        TFP=RRd3He*Wd3He+RRddp*Wddp+RRddn*Wddn
C         [W]
C         RRddn*Wddn/4. is fraction of DDn power to 3He, not n

C     steady state, fuel losses equal inputs
C     RRd3He... can be considered negligible compared to n/tauN, but include
C     to be accurate
        escrate =n*volplas/tauN
C         [s^-1]
        burnrate=RRd3He+RRddn+RRddp
C         [s^-1]
        lossrate=escrate+burnrate
C         [s^-1]
        mdfuel=lossrate*MWfuel
C         [kg/s]

      surf=(2.*pi*rcoil*Lcham + pi*(rcoil**2.-(Xrplug*rcoil)**2.)
     &+ pi*(rcoil**2.-(Xrnoz*rcoil)**2.))
     &/(2.*pi*rcoil*Lcham + 2.*pi*rcoil**2.)
C         fraction of surface area not "holes" (ends)

C     calculations of radiation sources
        c1=5.35e-31*fz*(kj*Zj**2.+kk*Zk**2.)
        d1=2.5e-32*fz**2.*(1.+Ti/(fz*Te))*(1.+Te/204.)
        Pbr=c1*n**2.*Te**0.5*volplas
C         [W]
```

```
        Pcy=d1*(1.-beta)/beta*n**2.*Te**2.*Kc*volplas
C         [W]

C   assume that all of reflected cyclotron radiation is absorbed by fuel
C   plasma and propellant: assume it is divided between fuel and prop
C   according to N
        refl=1.-Kc
        PradRfl=refl*surf*Pcy
C         [W]
        Pradplas=cycabs*PradRfl
C         [W]

C    plasma power balance: inputs=outputs (steady state)
C    using enter/exit control volume
C    Pinj+Pc=Plp+Pbr+(Pcy-Pradplas)
        Pinj=Plp+Pbr+(Pcy-Pradplas)-Pc
        IF (Pinj.LT.0) Pinj=0.
C    can't have negative injected energy - excess energy will go to
C    heat up plasma: what happens to plama temperature, particle loss rate
C    ?Examine.  Can hold at certain temp by varying fuel mixture

        Plp=(1.-y)*Pc + escrate*J_KeV*T
C         [W]
*****     END FRC PLASMA POWER BALANCE     *****

      END
C       subroutine pow_f
```

A.11 therm-f.for

```
      SUBROUTINE therm_f

      COMMON/...

*****      BEGIN FRC THERMAL POWER     *****
C     examine later if can ignore neutron heating of shield
C     else have to get into KERMAs (look at Steve Gruneisen paper)
        Prad=Pbr+Kc*Pcy

C   CALCULATE I^2R HEATING IN THETA PINCH, PLUG, AND NOZZLE
C     from Dolan p.611, tensile stress on solenoid=
C     eT=(B^2/2f0)(r1/(r2-r1)+1/3), drcoil=r2-r1,r1=rcoil
        drcoil=rcoil/(tensmax/Xtens*2.*mu0/Bext**2.-1./3.)
        alphg=(rcoil+drcoil)/rcoil
        betag=Lcham/(2.*rcoil)
        gab=(betag/(2.*pi*(alphg**2.-1.)))**0.5*
     &LOG((alphg+(alphg**2.+betag**2.)**0.5)/(1.+(1.+betag**2.)**0.5))
        lambda=0.9
        Pcoil=(Bext/(lambda*mu0*gab))**2.*resmat*rcoil/100.
C         rcoil/100. converts rcoil[cm] to rcoil[m]

C     assume plug area is square, thickness of plug
        Bplug=XBplug*Bext
        rplug=Xrplug*rcoil
        drplug=rplug/(tensmax/Xtens*2.*mu0/Bplug**2.-1./3.)
        alphg=(rplug+drplug)/rplug
        betag=dzplug/(2*rplug)
        gab=(betag/(2.*pi*(alphg**2.-1.)))**0.5*
     &LOG((alphg+(alphg**2.+betag**2.)**0.5)/(1.+(1.+betag**2.)**0.5))
        lambda=0.9
        Pplug=(Bplug/(lambda*mu0*gab))**2.*resmat*rplug/100.
C         rplug/100. converts rplug[cm] to rplug[m]

        Bnoz=XBnoz*Bplug
        rnoz=Xrnoz*rcoil
        drnoz=rnoz/(tensmax/Xtens*2.*mu0/Bnoz**2.-1./3.)
        alphg=(rnoz+drnoz)/rnoz
        betag=dznoz/(2.*rnoz)
      write(*,*)'rnoz,drnoz',rnoz,drnoz
      write(*,*)'alphg,betag',alphg,betag
      write(*,*)'(betag/(2.*pi*(alphg**2.-1.)))**0.5'
      write(*,*)(betag/(2.*pi*(alphg**2.-1.)))**0.5
        gab=(betag/(2.*pi*(alphg**2.-1.)))**0.5*
     &LOG((alphg+(alphg**2.+betag**2.)**0.5)/(1.+(1.+betag**2.)**0.5))
        lambda=0.9
        Pnoz=(Bnoz/(lambda*mu0*gab))**2.*resmat*rnoz/100.
C         rnoz/100. converts rnoz[cm] to rnoz[m]

        Preactor=Pcoil+Pplug
```

```
      Pth=Prad+Preactor+Pnoz

C          [W]
*****      END FRC THERMAL POWER      *****


      END
C       subroutine was_f
```

A.12 cooling.for

```
      SUBROUTINE cooling

      INCLUDE comblk.for

*****     BEGIN GENERIC COOLING, POWER GEN., Mix. CONDS.     *****
C   SET PROPELLANT FLOW VALVES TO COOL COMPONENTS, RUN GENERATOR
C   If sending all the propellant to be heated absorbs all the thermal
C   radiation power, only as much propellant will be sent that will allow the
C   highest temperature difference (materials limited) to make the generators
C   more efficient.
C   Similarly, if sending all heated propellant to generators produces too
C   much elec. power then only as much prop. as nec. will be sent;
C   otherwise the remainder is dumped in cold.


c***********NO RADIATOR
        mdprop=Pth/(hMat-h0)
        mdout=mdfuel+mdprop
C         [kg/s]
        gamCool=1
        w=1

        hmin=0.1*hmat
        gamGen=1
        wdot=gamGen*mdprop*(hMat-hmin)
        PelGen=etaGen*wdot
        PelReq=Ppayld+Preactor+Pnoz+Pinj/etaInj
C         [W]
        PextReq=PelReq-PelGen
        IF (PelGen.GT.PelReq) THEN
          PelGen=PelReq
          wdot=PelGen/etaGen
          gamGen=wdot/(mdprop*(hMat-hmin))
          PextReq=0
        ENDIF


C   solve for enthalpy of propellant mixing together again after cooling
C   nozzle, running generator, or being dumped in cold
        hprop=hMat*(1.-gamGen) + (hMat-hmin)*gamGen
        Pprop=mdprop*(hprop-h0)
C         [W]

C   Assume the plasma loss particles (with power Plp) and and the propellant
C   (with power Pprop) mix uniformly and come to a mixing temperature Tmix
C   (i.e. velocity=0).
        Pmix=Plp+Pprop
C       [W]
```

```
*****     END GENERIC COOLING, POWER GEN., Mix. CONDS.     *****


      END
C       subroutine cooling
```

A.13 saha.for

```fortran
      SUBROUTINE saha (U,temp,density,ionized)
      DOUBLE PRECISION dp1,dp2,dp3,dp4,Xsaha,ionized
      REAL EXP
      REAL U,temp,density


C       need to used double precision because of limited word-length of
C       IBM PC (1e+/-37)
C       n*1D6 converts n[cm^-3] to n[m^-3]
C       temp/11604.85 converts T[K] to T[eV]
C       U is required ionization energy, 13.6 eV for hydrogen

      dp1=(U/(temp/11604.85))
      dp2=(3.313e-28)*(density*1.e6)
      IF(temp.LT.0.) THEN
        write(*,*)'Saha: Not ignited'
        ionized=0
        RETURN
      ELSE
        dp3=(temp/11604.85)**(-3./2.)
      ENDIF

      IF(dp1.GT.70.) THEN
        ionized=0.
      ELSE
        dp4=EXP(dp1)
        Xsaha=dp2*dp3*dp4
        IF (Xsaha.EQ.0) THEN
          write(*,*)'Saha algorithm failed...'
          write(*,*)'dp1=(U/(temp/11604.85))=',dp1
          write(*,*)'dp2=(3.313e-28)*(density*1.e6)=',dp2
          write(*,*)'dp3=(temp/11604.85)**(-3./2.)=',dp3
          write(*,*)'dp4=EXP(dp1)=',dp4
          ionized=-1.
        ELSE
          ionized=((1.+4.*Xsaha)**0.5-1.)/(2.*Xsaha)
        ENDIF
      ENDIF


      END
C       subroutine saha
```

A.14 rocket.for

```
      SUBROUTINE rocket

      INCLUDE comblk.for

*****     BEGIN GENERIC ROCKET CALCS.     *****
C   THRUST AND SPECIFIC IMPULSE
C     Assume propellant particles enter into the throat of the magnetic
C     nozzle completely ionized; therefore the magnetic nozzle can direct
C     the propellant along with the plasma loss particles.

C       n_ions=n_fuel + *n_prop
        n_ions=(n*volplas+Xn*n*(volcham-volplas))/volcham
C         [cm^-3]
C       n_elec=n_fuel*Z_fuel + n_prop*Z_prop
        n_elec=(n*volplas*fz+Xn*n*(volcham-volplas)*Zprop)/volcham
C         [cm^-3]
        n_mix=(n_ions+n_elec)*volcham/volmix

        y_ions=n_ions/(n_ions+n_elec)
        y_elec=n_elec/(n_ions+n_elec)
C         mole fraction of ions,electrons
C       MWions=(N_fuel*MWfuel + N_prop*MWprop)/N_ions
        MWions=(n*volplas*MWfuel+Xn*n*MWprop*(volcham-volplas))
     &  /(n_ions*volcham)
C         [kg/ion (ave)]
        x_ions=y_ions*MWions/(y_ions*MWions + y_elec*MWelec)
        x_elec=y_elec*MWelec/(y_ions*MWions + y_elec*MWelec)
C         mass fraction of ions,electrons

C   now the power is divided among an electron gas and an ion gas.
C     R=8.3143 J/mol K
C     assuming an ideal gas, Cp=Cv + R
C     ie. Cp=R*(5./2.0)=20.786 J/mol K
C     but R/Nav=k=Boltzmann's Constant
        CpIon=5./2.*k/MWions
        CpElec=5./2.*k/MWelec

C     ions and electrons come to thermal equilibrium and all energy is
C     thermal (stagnation)

C       Pmix=(Cp*Tmix)mdout, solve for Tmix
        Tmix=Pmix/(mdout*(CpIon*x_ions +CpElec*x_elec))

C       Tmix/TTH p Xtemp w 1.35 AND VEXIT/VTHROAT p Xvel w 2.0
        TTH=Tmix/Xtemp


C   USE SAHA EQUATION TO CALCULATE DEGREE OF PROPELLANT IONIZATION
C   FOR MAGNETIC NOZZLE (assumes hydrogen propellant)
```

```
C   if ionization isn't near 100% mag. nozzle is not as effective

        CALL saha (Uion,Tmix,n_mix,ionized)

C   from conservation of energy: CP*DELTAT=(1/2)*VTHROAT**2
C   thermal energy is converted to enthalpy of the plasma
        VionTH=SQRT(2.*CpIon*(Tmix-TTH))
        VelecTH=SQRT(2.*CpElec*(Tmix-TTH))

C   flow exits Xvel times as fast because of expansion
C   through the nozzle.
        VionEX=Xvel*VionTH
        VelecEX=Xvel*VelecTH

        elecTHR=x_elec*mdout*VelecEX
        ionTHR=x_ions*mdout*VionEX
        IF(reactype.EQ.3)NFTHRST=0.
C         if FRC, no "non-pinch" thrust
        thrust=elecTHR + ionTHR + NFTHRST
        Isp=thrust/(g*mdout)

        IF(Isp*g*LOG(1./Xtank+1).LE.deltav)THEN
          WRITE(*,*)'ROCKET: Isp too low for requested delta V; ',
     &'Try higher energy.'
          bomb=1
C         prevent bomb in subsequent subroutines with SQRT(-..)
        ENDIF

*****     END GENERIC ROCKET CALCS.     *****


      END
C       subroutine rocket
```

A.15 mass.for

```
      SUBROUTINE mass

      INCLUDE comblk.for

*****     BEGIN SPECIFIC REACTOR MASS CALC     *****

      IF(reactype.EQ.2) THEN
*****     BEGIN DPF MASS CALC     *****
C    DETERMINE THE NECESSARY CAPACITOR MASS USING INPUTTED
C    SPECIFIC ENERGY
C    ASSUME EFFICIENCY OF CONVERSION FROM ELECTRICAL TO
C    MAGNETIC ENERGY IS CONSTANT.
C    W0=0.5*C0*V0**2=B**2/(2*mu0)
C    SINCE B IS PROPORTIONAL TO I, ASSUME NECESSARY STORED
C    ELECTRICAL ENERGY IS PROPORTIONAL TO I**2.
C    W=W0*(IOPT/IO)**2

C    NEED ELECTEN IN kJ AND specCap IN kJ/kg
        ELECTEN=0.5*CAP*VOLT**2.
c     *((IMOPT+IMAGNET)/IMAX)**2.
        Mcap=ELECTEN/1000./specCap
C      assume lanode and cathode are tubes with thickness 20% of radius
        reactor=Lanode*pi*rhomat*(Ranode**2.+Rcath**2.*(1.2**2.-1.))
C  take from dimensions, material density, etc
        injector=0.
*****     END DPF MASS CALC     *****
      ENDIF

      IF(reactype.EQ.3) THEN
*****     BEGIN FRC MASS CALC     *****
        theta=rhomat*Lcham*pi*((rcoil+drcoil)**2.-rcoil**2.)
C          [kg]
        plug=rhomat*dzplug*pi*((rplug+drplug)**2.-rplug**2.)
C          [kg]
        reactor=theta+plug
C      DPF only requires puff fill gas introduction system, not injector
C      assume injector mass scales linearly with mass flow to be injected
        injector=mdfuel*specInj
        Mcap=0.
*****     END FRC MASS CALC     *****
      ENDIF

*****     END SPECIFIC REACTOR MASS CALC     *****


*****     BEGIN GENERIC MASS CALC     *****

        mdout=mdfuel+mdprop
        ex=EXP(deltav/(Isp*g))-1
```

```
      tms=msys*ex/(mdout*(1-Xtank*ex))

      nozzle=rhomat*dznoz*pi*((rnoz+drnoz)**2.-rnoz**2.)
C        [kg]
C        assumes nozzle has rectangular cross section
C        dznoz * drnoz

C     shielding from parasitic DDn
C     assumes constant flux over tms, ending with dose=maxdose
C     set nflux allowable by max dose
      nflux=maxdose/(tms*remnflux)
C        [cm^-2 s^-1]
      sigeff=rhoLiH/MWLiH*(15.+1.)*1.e-24
C        sigeff=[cm^-2] effective e of LiH shield material
C        1.e-24 converts barns to cm^2
C     assume nflux is attenutated as nflux=nflux0*EXP(-sigeff*drshld)
C     solve for drshld
      nflux0=RRddn*volplas*Xcone/(4.*pi*rpssngr**2.)
C        [s^-1]
      drshld=1./sigeff*LOG(nflux0/nflux)
C        [cm]
      r2=r1+drshld
C        [cm]
      volshld=4./3.*pi*(r2**3.-r1**3.)*Xcone
      shield=rhoLiH*volshld
C        [kg]

      radiator=(1.-w)*Pth*specRad

C     assume generator mass scales linearly with power required
C     assume 2 specific energies: generators, and external supply
      powersys=specGen*PelGen+specExt*PextReq+Mcap


      msys=injector+shield+radiator+powersys+Mcap+reactor+nozzle+
     &payload+shelter


C     mass of fuel, propellant + tanks + pumps, etc.
c    deltav will ultimately come from mission analysis program

      fuel=tms*mdfuel
      prop=tms*mdprop
      tanks=Xtank*(fuel+prop)
      mf=msys+tanks
      m0=mf+prop+fuel
*****    END GENERIC MASS CALC     *****

      END
C     subroutine mass
```

A.16 output.for

```
C23456789                                                          0
      SUBROUTINE output

      INCLUDE comblk.for

************ OUTPUT ******************

      tanks=Xtank*(prop+fuel)
      tmd=tms/(24.*3600.)
C       tmd[d]*3600.*24.=tms[s]


C   display output values
      IF(loglin.EQ.1)THEN
C       increment IncVar LOG10
        WRITE(*,92)
     &  '+ heps',count/howmany*100,'%  ',IncVar,'=',10**operate
      ELSE
C       otherwise increment IncVar linearly
        WRITE(*,92)'+ heps',count/howmany*100,'%  ',IncVar,'=',operate
      ENDIF


      IF(o1)WRITE(55,191)' T[keV]=',T

      IF(o2)THEN
        IF(.NOT.o9) THEN
          WRITE(55,*)
          WRITE(55,*)'PLASMA PARAMETERS'
          IF(reactype.EQ.2)THEN
            WRITE(55,191)' Bnoz[T]=',Bnoz
            WRITE(55,191)' pulse voltage[V]=',volt
            WRITE(55,191)' pulse current[A]=',Imax
          ELSEIF(reactype.EQ.3)THEN
            WRITE(55,192)' Bnoz, Bplug[T]=',Bnoz,',',Bplug
            WRITE(55,192)' Bext, Bint [T]=',Bext,',',Bint
            WRITE(55,191)' beta =',beta
            WRITE(55,191)' tauN [s] =',tauN
          ENDIF
        ENDIF


        IF (Pinj.EQ.0) THEN
          IF(o9)THEN
            WRITE(55,131)T
          ELSE
            WRITE(55,191)' Qp,rpPf/Pi= 1  *ignited*'
          ENDIF
        ELSE
```

```
          IF(o3)THEN
            WRITE(55,132)T,TFP/Pinj
          ELSE
            WRITE(55,191)' Qp,rpPf/Pi=',TFP/Pinj
          ENDIF
        ENDIF
      ENDIF

C     fuel + propellant masses
      IF(o3)THEN
        IF(o9)THEN
          WRITE(55,134)T,mdout,prop,fuel
        ELSE
          WRITE(55,*)
          WRITE(55,*)'EXHAUST MASSES'
          WRITE(55,191)' mdout[kg/s]=',mdout
          WRITE(55,191)' prop[kg]+sys   =',prop
          WRITE(55,191)' fuel[kg]+sys   =',fuel
        ENDIF
      ENDIF

C     component masses
      IF(o4)THEN
        IF(o9)THEN
          WRITE(55,139)T,injector,radiator,shield,reactor,nozzle,
     &powersys,payload,shelter
        ELSE
          WRITE(55,*)
          WRITE(55,*)'COMPONENT MASSES'
          WRITE(55,191)' fuel inj.[kg]  =',injector
          WRITE(55,191)' tanks[kg]      =',tanks
          WRITE(55,191)' radiator[kg]   =',radiator
          WRITE(55,191)' shield[kg]     =',shield
          WRITE(55,191)' reactor[kg]    =',reactor
          WRITE(55,191)' nozzle[kg]     =',nozzle
          WRITE(55,191)' power sys.[kg] =',powersys
          WRITE(55,191)' payload[kg]    =',payload
          WRITE(55,191)' shelter[kg]    =',shelter
          WRITE(55,*)   '----------------------------'
          WRITE(55,191)' final mass [kg]=',mf
        ENDIF
      ENDIF

C     output powers (heat)
      IF(o5)THEN
        IF(o9)THEN
          WRITE(55,134)T,Pradwall,Pcoil,Pnoz
        ELSE
          WRITE(55,*)
          WRITE(55,*)'THERMAL OUTPUT POWERS (HEAT)'
          WRITE(55,191)' Pradwall     [W]=',Pradwall
```

```
          WRITE(55,191)' Preactor     [W]=',Pcoil
          WRITE(55,191)' Pnoz         [W]=',Pnoz
          WRITE(55,*)    '----------------------------'
          WRITE(55,191)' Pth          [W]=',Pth
        ENDIF
      ENDIF

C     input powers (electrical)
      IF(o6)THEN
        IF(o9)THEN
          WRITE(55,136)T,Ppayld,Pinj/etaInj,Pcoil,Pplug,Pnoz
        ELSE
          WRITE(55,*)
          WRITE(55,*)'ELECTRICAL INPUT POWERS'
          WRITE(55,191)' Ppayld   [W]=',Ppayld
          WRITE(55,191)' Pinj/oinj[W]=',Pinj/etaInj
          WRITE(55,191)' Pcoil    [W]=',Pcoil
          WRITE(55,191)' Pplug    [W]=',Pplug
          WRITE(55,191)' Pnoz     [W]=',Pnoz
          WRITE(55,*)    '----------------------------'
          WRITE(55,191)' PelReq   [W]=',PelReq
        ENDIF
      ENDIF

C     other powers (electrical)
      IF(o7)THEN
        IF(.NOT.o9)THEN
          WRITE(55,*)
          WRITE(55,*)'GENERATED AND EXTERNALLY REQUIRED ELEC. POWER'
          WRITE(55,191)' PelGen   [W]=',PelGen
          WRITE(55,191)' PextReq  [W]=',PextReq
          WRITE(55,191)' gamGen=',gamGen
        ENDIF
        IF (PextReq.EQ.0) THEN
          IF(o9)THEN
            WRITE(55,131)T,gamGen
          ELSE
            WRITE(55,191)' Qp,spPelGen/PextReq= 1  *ignited*'
          ENDIF
        ELSE
          IF(o9)THEN
            WRITE(55,133)T,gamGen,PelGen/PextReq
          ELSE
            WRITE(55,191)' Qp,spPelGen/PextReq=',PelGen/PextReq
          ENDIF
        ENDIF
      ENDIF

C     rocket output data
      IF(o8)THEN
        IF(o9)THEN
```

```
        WRITE(55,134) T, thrust, NFTHRST, thrust/NFTHRST

      ELSE
        WRITE(55,*)
        WRITE(55,*)'RAW ROCKET OUTPUT DATA'
        WRITE(55,192)' Tmix[K],Tmix[eV]=',Tmix,',',Tmix/11604.85
        WRITE(55,191)' n_mix[cm^-3]=',n_mix
        WRITE(55,191)' Degree of propellant ionization =',ionized
        IF (ionized.LT.0.3) WRITE(55,*)'  * Not ionized enough for ',
     &'magnetic nozzle *'
        IF (Tmix.LT.2000) WRITE(55,*)'  * use normal nozzle *'
        WRITE(55,192)' Thrust[N],Thrust[lbf]=',thrust,',',thrust/
     &4.4482
        WRITE(55,191)' Specific Impulse[s]=',Isp
        WRITE(55,191)' Mission firing time [d]=',tmd
        WRITE(55,192)' M0,Mf [kg]',m0,',',mf
        WRITE(55,*)
        WRITE(55,*)'ROCKET FIGURES OF MERIT'
        WRITE(55,191)' Initial Thrust/weight   =',thrust/(m0*g)
        WRITE(55,191)' Jet Power w/o fusion [W]=',
     &NFTHRST**2/(mdfuel*2)
        WRITE(55,191)' Jet Power w/  fusion [W]=',g/2.*thrust*Isp
        WRITE(55,191)' Fus. Chrg. Part. Pow. W]=',TFP
        WRITE(55,191)' Jet Specific Power '0 [kg/kWjet]=',
     &m0/(g/2.*thrust*Isp/1.e3)
        WRITE(55,191)' Overall Efficiency Pjet/Pf=',
     &(g/2.*thrust*Isp)/TFP
        WRITE(55,191)' Rocket Eq. Delta Vee [m/s]=',deltav
        WRITE(55,191)' Payload mass/System mass=',payload/m0
        WRITE(55,191)' Mrpmdprop/mdfuel=',mdprop/mdfuel


      ENDIF
     ENDIF

     IF(.NOT.o9) THEN
       WRITE(55,*)
       WRITE(55,*)'*****     Next temperature increment     *****'
       WRITE(55,*)
     ENDIF

C    varitop input file
c     IF(o10)THEN
c       write(*,*)
c       write(*,191)' p0[kw,jet]=',g/2.*thrust*Isp/1.e3
c       write(*,191)' m0[kg]=',m0
c       write(*,191)' is[s]=',Isp
c       write(*,191)' alfa1[kg/kw]=',mf/(g/2.*thrust*Isp/1.e3)
c       write(*,191)' alfa2[kg]=',m0-mf
c       write(*,191)' tend[d]=',tmd
c       write(65,191)' p0=',g/2.*thrust*Isp/1.e3
```

```
c        write(65,191)' m0=',m0
c        write(65,191)' is=',Isp
c        write(65,191)' alfa1=',mf/(g/2.*thrust*Isp/1.e3)
c        write(65,191)' alfa2=',m0-mf
c        write(65,191)' tend=',tmd
c      ENDIF

      IF(o12)WRITE(55,134)
     &T,deltav,tmd,payload/m0

c        Pprop=Pmix-Plp
      IF(o13)WRITE(55,136)
     &T,NFTHRST**2/(mdfuel*2),Plp-NFTHRST**2/(mdfuel*2),Pmix-Plp,
     &Pmix,g/2.*thrust*Isp

      IF(o14)WRITE(55,135)
     &T,NFTHRST**2/(mdfuel*2)/Pmix,(Plp-NFTHRST**2/(mdfuel*2))/Pmix,
     &(Pmix-Plp)/Pmix,g/2.*thrust*Isp

      IF(o15)WRITE(55,136)
     &T,g/2.*thrust*Isp,PjetNF,g/2.*thrust*Isp/PjetNF,
     &Pinj,g/2.*thrust*Isp/Pinj

      END
C        subroutine output
```

A.17 reactor.hep

```
reactype:2=Perform DPF simulation,3=Perform FRC simulation
2.
IncVar: indep., incremented var.: T,deltav,..
deltav
howmany = points to graph between lower and upper values of IncVar
10
lower = lower value of IncVar
5e2
upper = upper value of IncVar
1.5e4
loglin= LOG10 or linear increment of IncVar (1=LOG10,other=linearly)
1.
T = [keV] Plasma thermal energy T=Ti+Te
150.
xprot = fuel particle density fraction of protons
0.
xdeut =
0.50
xtrit =
0.
x3He  =
0.50
x6Li  =
0.
x11B  =
0.
xalfa =
0.
rt = unitless, ratio T(electron)/T(ion)
1.
Kc = unitless,fraction of cyclotron radiation absorbed in wall
0.1
cycabs = fraction of cyclotron radiation absorbed in plasma
0.6
y = unitless,frac. charged part. power absorbed in plasma
0.8
Xn = Ratio of fuel density to propellant density in reaction chamber
1.
etaGen = generator efficiency (thermal->electrical)
0.3
r1[cm] = inner radius of shield
100.
rpssngr[cm] = passenger distance from source
1000.
Xcone = fraction of isotropically radiated neutrons to be shielded against
0.125
specRad[kg/Wth]= specific mass of radiators (620 kg/MWth)
0.621e-3
specGen[kg/Wel]= specific mass of generators
```

```
2.e-4
specExt[kg/Wel]= specific mass of external electrical eng. supply
3.e-4
resmat[ohm m]=resistivity of theta pinch/mirror materials (Cu=1.72e-8)
1.72e-8
rhommat[kg/cm^3]=density of theta pinch/mirror materials (Al=2.67g/cm^3)
8.9e-3
tensmax[Pa]=tensile strength of theta magnetic materials(280e6=Cu) w/cables,etc.
280.e8
Itens=ratio max tens. strenth of material to tensile stress (safety factor)
2.
maxdose [cSv] over mission travel time (assuming reactor sole source)
5.
ddnspin = suppressing/enhancing factor from spinpolarization (1=no effect)
1.
ddpspin = suppressing/enhancing factor from spinpolarization
1.
d3Hespin = suppressing/enhancing factor from spinpolarization
1.
```

A.18 output.hep

```
o0:WRITE Initial values (T,t=yes, F,f=no)
F
o1:WRITE Ti[keV]
F
o2:WRITE plasma parameters
F
o3:WRITE exhaust
F
o4:WRITE component masses
F
o5:WRITE waste output powers (heat)
F
o6:WRITE electrical input powers
F
o7:WRITE generated and externally required elec. power
F
o8:WRITE Rocket output data
F
o9:WRITE data in table form (without text explainations)
T
o10:WRITE (was VARITOP)
F
o11:
F
o12:T, deltaV, tmd, payload/m0
T
o13:T,mdfuel*Vrun**2/2,Plp-mdfuel*Vrun**2/2,Pprop,Pmix,Pjet
F
o14:T,PjetNF/Pmix,Pc/Pmix,Pprop/Pmix,Pjet
F
o15:T,Pjet,Pjet w/o, ratio
F
*************************************************************

Key for table form output: (incomplete)
o1:
o2: T, Qp
o3: T,mdout,prop,fuel
o4: T,injector,radiator,shield,reactor,nozzle,powersys,payload,shelter
o5: T,Pradwall,Pcoil,Pnoz
o6: T,Ppayld,Pinj/etaInj,Pcoil,Pplug,Pnoz
o7: T,TFP/Pinj
o8: T, NFTHRST, thrust, thrust/NFTHRST, Isp,
o11:T, (NFTHRST**2/(mdfuel*2)), g/2.*thrust*Isp,
     &(g/2.*thrust*Isp) / (NFTHRST**2/(mdfuel*2)),
     &(g/2.*thrust*Isp)/TFP
o12:T, deltaV, tmd, payload/m0
Plp=TDELTAP+mdfuel*Vrun**2/2
o13:T,mdfuel*Vrun**2/2,Plp-mdfuel*Vrun**2/2,Pprop,Pmix,Pjet
```

```
o14:T,PjetNF/Pmix,Pc/Pmix,Pprop/Pmix,Pjet
o15:T,Pjet,Pjet w/o, ratio
```

A.19 dpf.hep

```
RANODE=ANODE RADIUS [cm]
5.08
RCATH=CATHODE RADIUS [cm]
8.00
LANODE=ANODE LENGTH [cm]
38.2
RHOION=INITIAL FILL GAS DENSITY [kg/cm^3]
2.2E-10
VOLT=CAPACITOR BANK CHARGING POTENTIAL IN VOLTS
27000.
CAP=CAPACITANCE IN FARADS
3.55E-4
LINIT=INITIAL CAPACITANCE
2.5E-8
FSNPLW=SNOWPLOW EFFICIENCY FACTOR
0.7
FPNCH=PINCH EFFICIENCY FACTOR
0.25
LPNCH=PINCH LENGTH [cm]
2.54
PNCHRAD=PINCH RADIUS [cm]
1.5E-1
REPRATE=PLASMA FOCUS FIRING RATE IN S**-1
100.
PNCHTIM=DURATION OF STABLE PINCH IN SECONDS
1.0E-4
ITERS=NUMBER OF ITERATIONS
500.
DSCHRG=duration of discharge [s]
1.0E-7
IMAGNET=magnetic current [A]
3.18E5
Bnoz=[T]
8.2e3
specCap=CAPACITOR BANK SPECIFIC ENERGY IN KJ/KG
2.0
Xrnoz=ratio nozzle radius/coil radius
0.1
specInj[kg/Wmdfuel]= specific mass of fusion fuel injectors(dpf=0,gas puff)
0.
etaInj=Injector efficiency (electrical->injected)
1.
```

A.20 rocket.hep

```
Mr = Ratio of propellant mass flow to fuel mass flow
1.e3
Zprop = unitless,atomic number of propellant
1.
Aprop = unitless,atomic mass of propellant
1.
h0 = [J/kg] initial enthalpy of propellant (read from chart) (50 K)
1.403e8
hMat = [J/kg] enthalpy of propellant at materials-limited temperature (2000K)
1.716e8
Xtemp = Ratio of mixing temperature (absolute) to throat temperature (abs)
1.35
Xvel = Ratio of exit velocity to throat velocity of magnetic nozzle
2.
deltav [m/s] = mission deltav
3e3
payload[kg]= mass of desired payload
1.e5
Ppayld[W]= electrical requirements of payload
40.e3
shelter = [kg] mass of Solar Particle Event (SPE) storm shelter
5.5e3
dznoz=[cm]nozzle thickness
5.
volmix=[cm^3] mixing chamber volume
1.e7
Xtank=tank fraction, determine mass of tanks, feed lines, etc.
0.1
Tmix[K]=initial guess for exhaust mixing temperature
7.e3
```

A.21 tm.for

```
      program tm

      implicit real (a-z)
      CHARACTER pause*1

      DATA pi,Wd3He,grav/3.1415926,18.3,9.8/

131   FORMAT (1PE11.3)
132   FORMAT (1P2(E11.3))
133   FORMAT (1P3(E11.3))
134   FORMAT (1P4(E11.3))
135   FORMAT (1P5(E11.3))
137   FORMAT (1P7(E11.3))
138   FORMAT (1P8(E11.3))
141   FORMAT (A,1PE11.3)
142   FORMAT (A,1P2(E11.3))
143   FORMAT (A,1P3(E11.3))
144   FORMAT (A,1P4(E11.3))
145   FORMAT (A,1P5(E11.3))
147   FORMAT (A,1P7(E11.3))
148   FORMAT (A,1P8(E11.3))

      Pf(kT) = inven*inven/Vp/4.*Wd3He*SVd3He(kT)*1.602e11 !MJ/us
      Pbr(kT) = 2.5*inven*inven/Vp*SQRT(kT)*5e-7 !MJ/us
      g(kT) = (Um*1e-3*Im + Pf(kT) - Pbr(kT))/(inven*1.602e-4) !keV/us

      OPEN (UNIT=45, FILE='tmi.dat', STATUS='OLD')
      READ(45,*)
      READ(45,*)W
      READ(45,*)
      READ(45,*)Idot0
      READ(45,*)
      READ(45,*)Ldot
      READ(45,*)
      READ(45,*)inven
      READ(45,*)
      READ(45,*)rc,ra
      READ(45,*)
      READ(45,*)nu
      READ(45,*)
      READ(45,*)xtank
      READ(45,*)
      READ(45,*)mpay
      READ(45,*)
      READ(45,*)deltav
      CLOSE (UNIT=45)

      write(*,141)' W              (MJ)',W
      write(*,141)' Idot0       (MA/us)',Idot0
```

```
write(*,141)' Ldot         (mH/s)',Ldot
write(*,141)' inventory N (*1e18)',inven
write(*,142)' rc,ra          (cm)',rc,ra
write(*,141)' reprate nu     (Hz)',nu
write(*,141)' xtank              ',xtank
write(*,141)' mpay           (kg)',mpay
write(*,141)' deltav       (km/s)',deltav
WRITE(*,*)'Press <Enter> to continue'
READ(*,'(A1)')pause

speccap=2e-3
mcap=W/speccap
write(*,141)' mcap (kg)',mcap
AU=(W*Idot0*Ldot*Ldot)**(1./3.)*1e1
AI=(W*Idot0/Ldot)**(1./3.)*1e1
At=(W/(Idot0*Idot0*Ldot))**(1./3.)*1e1
write(*,143)' AU(kV),AI(MA),At(us)',AU,AI,At
Um=0.64*AU
Im=0.64*AI
write(*,144)' U0,Um(kV),Im(MA),trise(us)',2.12*AU,Um,Im,1.43*At
tau_p=0.268*At
write(*,141)' tau_p (us)',tau_p
lp=rc-ra !cm
kT0=Im*Im*lp/(2*inven*1.602e-1)
write(*,141)' kT0 (keV) ',kT0
write(*,141)' SV(kT0)(cc/s)',SVd3He(kT0)
kr=70.
r0=ra/kr
Vp=pi*r0*r0*lp !cm^3
dens=inven*1e18/Vp !cm^-3
write(*,143)' r0(cm),Vp(cc),dens(/cc)',r0,Vp,dens
write(*,143)' UmIm,Pf,Pbr (MJ/us)',Um*1e-3*Im,Pf(kT0),Pbr(kT0)
Pfsv=inven*inven/Vp/4.*Wd3He*1.602e11
PbrkT=2.5*inven*inven/Vp*5e-7
write(*,141)' Pf(kT0)/<sv>      =',Pfsv
write(*,141)' Pf(kT0)/<sv>/N    =',Pfsv/(inven*1.602e-4)
write(*,141)' Pbr(kT0)/kT^1/2   =',PbrkT
write(*,141)' Pbr(kT0)/kT^1/2/N =',PbrkT/(inven*1.602e-4)
h=tau_p !us
g1=g(kT0)
g2=g(kT0+h/2.*g1)
g3=g(kT0+h/2.*g2)
g4=g(kT0+h*g3)
write(*,143)'kT1,kT2,kT3',kT0+h/2.*g1,kT0+h/2.*g2,kT0+h*g3
grk = (g1+2.*g2+2.*g3+g4)/6.
kTf = kT0 + grk*h
write(*,145)' g1,g2,g3,g4,grk',g1,g2,g3,g4,grk
kTave = kT0 + grk*h/2.
write(*,142)' kTf,kTave (keV)',kTf,kTave
Ef = tau_p*Pf(kTave) !MJ
Ebr = tau_p*Pbr(kTave) !MJ
```

```
      write(*,144)' Ef,Ebr (MJ), Qf,Qbr',Ef,Ebr,Ef/W,Ebr/W
      Nf_N = (2 * Ef)/(Wd3He*inven*1.602e-1)
      write(*,141)' Nf/N ',Nf_N
      IF (Nf_N.GT.1.0) write(*,141)' Qmax=',inven*Wd3he/W*1e-1
      write(*,*)'nu?'
      read(*,*)nu
      Pth = (W + Ef + Ebr)*nu !MW
      deltah = 1.05e2 !MJ/kg
c      deltah = 31.3 !MJ/kg using old 1.716-1.403 e8J/kg
      mdot = Pth/deltah !kg/s
      write(*,142)' Pth(MW),mdot(kg/s)',Pth,mdot
      Pprop = Pth - W*nu/0.5 !MW
      Vex = SQRT(2*Pprop/mdot)  !km/s
      write(*,142)' Pprop(MW),Vex(km/s)',Pprop,Vex
      kTprop=Prop/mdot*1.67e-2/1.602
      write(*,141)' kTprop,kTthr (eV)',kTprop,kTprop/1.35
      thrust=mdot*Vex
      write(*,142)' F(kN),Isp(*10^3 s)',thrust,Vex/grav
      write(*,*)'Deltav,max (km/s)',vex*LOG(1+1./xtank)
c      write(*,*)'Delta v?'
c      read(*,*)deltav
      ms=mcap+mpay
      ee=EXP(deltav/vex) - 1
      tmiss=ms/mdot*ee/(1-xtank*ee) !s
      tmd=tmiss/8.640e4
      write(*,141)' tmiss (d)',tmd
      m0=ms+mdot*tmiss*(1+xtank)
      alpha=m0/(Pprop*1e3)
      write(*,143)' m0(kg),mpay/m0,alpha(kg/kW)',m0,mpay/m0,alpha
      f_w=thrust*1e3/(m0*grav)
      write(*,141)' F/w0',f_w
      OPEN (UNIT=46, FILE='tmo.dat', STATUS='OLD')
      write(46,137)mpay,deltav,nu,tmd,f_w,alpha,mpay/m0
      end



C  for 3He(d,p)' rxn; cm^3/s
      REAL FUNCTION SVd3He(T)
      REAL a,b,c,d,T,x,LOG10
        x=LOG10(T)
        a=  0.3535589
        b= -3.310354
        c= 10.10471
        d=-25.67344
        SVd3He=10**(a*x**3+b*x**2+c*x+d)
      END
C     function SVd3He
```

A.22 tmi.dat

```
W (MJ) = ?"
100
Idot0 (MA/us) = ?"
10
Ldot (mH/s) = ?"
1
inven (*1e18) = ?"
100
rc,ra (cm) = ?"
8 3
reprate nu (s^-1)"
1
xtank=?"
.1
mpayload(kg)
1e5
deltav (km/s)
25
```

B c source code

B.1 cir.c

```
#include<math.h>
#include<stdio.h>

#define mu0 1.2566E-6
#define pi 3.141592654
#define J_MeV 1.60219E-13
#define J_keV 1.60219E-16
#define Wddn 3.27
#define Wddp 4.03
#define Wd3He 18.3

FILE *fp,*fq;
float time[1000],cv[1000],i[1000],find_max();
int max_records;
void graph(),read_data(),enter_variables(),*cross;
char xtype[20],ytype[20],ans;
float x_data[1000],y_data[1000];

float A[11]; /* polynomial solution coefficients */
/* C arrays start at 0 in references, but not in definitions */
void enter_variables(),description(),rundown();
float Ustar(),Istar(),SVd3He();
float t0,tf,tdivs,W,Idot0,Ldot;
float U0,t,h,Uopt,Imopt,topt,tmax,Imax,uuopt,Umax,L0,Lmax,
      Cap,Omega0,nopt,lambda,Rtot,tau,Omega, Qmax,Bcoeff,TFE,TEBrem,
      deut,hel3,RRddn,RRddp,RRd3He,dPFddn,dPFddp,dPFd3He,dEftot,dEbrem,
      volplas,kT_keV;
int test;
int j;
float Wrundown,W_pinch,t_pinch;
float kT,inventory,Zeff,r_0,r_anode,r_cathode,r_pinch,
      compression_ratio,vol_pinch,density,P_brem,P_fusion,
      E_brem,E_fusion,Q;
float AU,AI,At,tstar,U0star,Imstar,Umstar,tmstar;
float current,voltage;


main()
{
int first_time=1;
do
      {
      clrscr();
      printf("DPF Circuit Program\n\n");
      if (first_time) description();
      first_time=0;
      rundown();
      printf("\n\nDO YOU WISH TO ALTER DATA AND COMMENCE ANOTHER RUN <Y/N> ");
      }
```

```
while(ans=getch()=='y' || ans=='Y');
clrscr();
} /* main */


void rundown()
{
float temp;
float vrd;

enter_variables();

vrd=Ldot*2*pi/(mu0*log(r_cathode/r_anode));

AU=pow((W*Idot0*Ldot*Ldot),(1./3.));
AI=pow((W*Idot0/Ldot),(1./3.));
At=pow((W/(Idot0*Idot0*Ldot)),(1./3.));

/*
Uopt  = 2.12 * AU;
Imopt = 0.64 * AI;
topt  = 1.5  * At;
*/

U0star=2.12;
U0=AU*U0star;

/* GET COEFFICIENTS TO SERIES SOLUTION */
/* U*=SUM A[j]t*^j */
A[0]=U0star;
A[1]=0.;
for (j=0;j<=8;j++)
      {
      A[j+2] = -A[j]*U0star/(2*(j+1)*(j+2))
               -A[j+1]*(j+1)/((j+2)*U0star);
      }

if((fp=fopen("cir_out.dat","wt"))==NULL)
      {
      printf("Whoa! Cannot open output file: cir_out.dat\n");
      exit(1);
      }

/* find maximum current, time at max current */
/* RUNDOWN PHASE */
Imax=0;
tmax=0;
/* define timestep */
h=(tf-t0)/(tdivs-1);
/* use while loop here */
Wrundown=0;
```

```
tstar=t0/At;

while (Ustar(tstar)>0)  /* quit if voltage drops below 0 */
      {
      voltage=AU*Ustar(tstar);
      current=AI*Istar(tstar);
      t=At*tstar;
      if (current>Imax)
            {
            /* get the initial conditions for pinch phase */
            tmax=t;
            Imax=current;
            Umax=voltage;
            Wrundown+=voltage*current*h;
            if (t>tf) printf("Need longer rundown time\n");
            }
      fprintf(fp,"%11.3e %11.3e %11.3e\n",t,voltage,current);
      tstar+=h/At;
      }
fclose(fp);

if (Wrundown <= W)
      {
      W_pinch=W-Wrundown;
      }
else  {
      printf("No current maximum: taking W_pinch=0.1W\n");
      W_pinch=0.1*W;
      Imax=AI*Istar(0.9*tmax);
      Umax=AU*Ustar(0.9*tmax);
      }
t_pinch = W_pinch/(Imax*Umax);

kT=W_pinch/(inventory*J_keV);             /*keV*/
r_0=r_anode;
r_pinch=r_0/compression_ratio;            /*take r0=ra*/
vol_pinch=pi*r_pinch*r_pinch*(r_cathode-r_anode);
      /*take lpinch=rc-ra, experimental scaling*/
density=inventory/vol_pinch;
P_brem=density*5e-37*Zeff*density*sqrt(kT);
P_fusion=density/4.*SVd3He(kT)*density*1e-6*Wd3He*J_MeV;
E_brem=P_brem*vol_pinch*t_pinch;
E_fusion=P_fusion*vol_pinch*t_pinch;
Q=E_fusion/W;

printf("\n");
printf("vrd        (m/s)%11.3e \n",vrd);
printf("l_anode      (m)%11.3e \n",vrd*tmax);
printf("U0*,I*m,t*m     %11.3e%11.3e%11.3e \n",U0/AU,Imax/AI,tmax/At);
printf("tmax,tf      (s)%11.3e%11.3e \n",tmax,tf);
printf("Imax         (A)%11.3e \n",Imax);
```

```
printf("U0,Umax     (V)%11.3e%11.3e \n",U0,Umax);
printf("W,Wrundown  (J)%11.3e%11.3e \n",W,Wrundown);
printf("Wrd/W          %11.3e \n",Wrundown/W);
printf("t_pinch     (s)%11.3e \n",t_pinch);
printf("\n");
printf("kT            (keV)%11.3e\n",kT);
printf("Peak Brem.      (¿)%11.3e\n",kT/6.20);
printf("r_0,r_pinch     (m)%11.3e,%11.3e \n",r_0,r_pinch);
printf("density      (m^-3)%11.3e \n",density);
printf("vol_pinch     (m^3)%11.3e \n",vol_pinch);
printf("<ev>       (cm^3/s)%11.3e\n",SVd3He(kT));
printf("P_brem,P_fusion (W)%11.3e%11.3e\n",P_brem,P_fusion);
printf("E_brem,E_fusion (W)%11.3e%11.3e\n",E_brem,E_fusion);
printf("Q,E_brem/W         %11.3e%11.3e\n",Q,E_brem/W);

if((fq=fopen("cir_io.dat","wt"))==NULL)
      {
      printf("Whoa! Cannot open output file: cir_io.dat\n");
      exit(1);
      }
fprintf(fq,"%10.3e%10.3e%10.3e%10.3e%10.3e%10.3e%10.3e%10.3e",
W,Idot0,Ldot,r_cathode,r_anode,inventory,Zeff,compression_ratio);
fprintf(fq,"%10.3e%10.3e%10.3e%10.3e%10.3e%10.3e%10.3e%10.3e%10.3e\n",
Imax,t_pinch,r_pinch,kT,density,E_fusion,E_brem,Q);
fclose(fq);


printf("\nDO YOU WISH TO GRAPH DATA <Y/N>");
if (ans=getch()=='y'|| ans=='Y')
      {
      read_data();
      test=menu();
      while(test!=0)
            {
            graph();
            test=menu();
            }
      }
 } /* rundown */

void description()
{
printf("program calculates capacitor voltage, current vs. time        \n");
printf("for simplified rundown phase for DPF circuit.                  \n");
printf("capacitor charge = series solution to RLC circuit, L=L0+Ldot*t \n");
printf("q=SUM Ai*t**i                                                  \n");
printf("10th order polynomial                                          \n");
printf("program author: R. Nachtrieb, Nov91                            \n");
printf("last modified:               Mar92                           \n\n");
}
```

```
float Ustar(t)
float t;
{
float voltage;
voltage = A[0] + A[1]*t + A[2]*t*t + A[3]*t*t*t +
A[4]*pow(t,4) + A[5]*pow(t,5) + A[6]*pow(t,6) + A[7]*pow(t,7) +
A[8]*pow(t,8) + A[9]*pow(t,9) + A[10]*pow(t,10);
return (voltage);
}

float Istar(t)
float t;
{
float current;
current = -2./(U0star*U0star)*
(A[1] + 2*A[2]*t + 3*A[3]*t*t + 4*A[4]*t*t*t + 5*A[5]*pow(t,4) +
6*A[6]*pow(t,5) + 7*A[7]*pow(t,6) + 8*A[8]*pow(t,7) + 9*A[9]*pow(t,8) +
10*A[10]*pow(t,9));
return (current);
}
```